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## IT CookBook, 실감나게 배우는 제어공학 : 핵심 개념부터 응용까지

### [연습문제 답안 이용 안내]

- 본 연습문제 답안의 저작권은 한빛아카데미(주)에 있습니다.
- 이 자료를 무단으로 전제하거나 배포할 경우 저작권법 136조에 의거하여 최고 5년 이하의 징역 또는 5천만원 이하의 벌금에 처할 수 있고 이를 병과(併科)할 수도 있습니다.

## Chapter 01 연습문제 답안

1.1 ④

1.2 ②

1.3 ③

1.4 ①

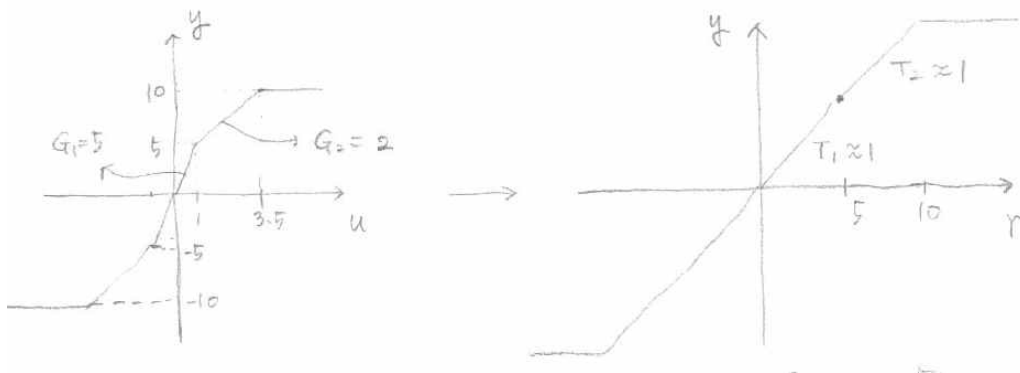
1.5 ②

1.6 ①

1.7  $vc = e^{-\frac{5}{RC}t}$   
 $e^{-\frac{5}{RC}t} = 0.1, t = \frac{RC}{5} \ln 10 = \frac{10^{-6}}{5} \ln 10 \leftarrow \text{더 빨리 수렴}$   
 $vi = -2vc$ 인 경우,  $vc = e^{-\frac{3}{RC}t}, t = \frac{RC}{3} \ln 10 = \frac{10^{-6}}{3} \ln 10$

$vi = -4vc$ 인 경우,  $vc = e^{-\frac{5}{RC}t}$   
 $vi = -2vc$ 인 경우,  $vc = e^{-\frac{3}{RC}t}$

1.8 ①



1.9 ①

1.10 ①

1.11 ①

$$1.12 \quad \frac{\pi}{w} \frac{-w^2 + 6}{(-w^2 + 6)^2 + 25w^2}$$

$$1.13 \quad vi = Kv_c$$

$$1.14 \quad v(t) = -8 \frac{d\chi(t)}{dt} - 4\chi(t)$$

$$1.15 \quad \chi(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

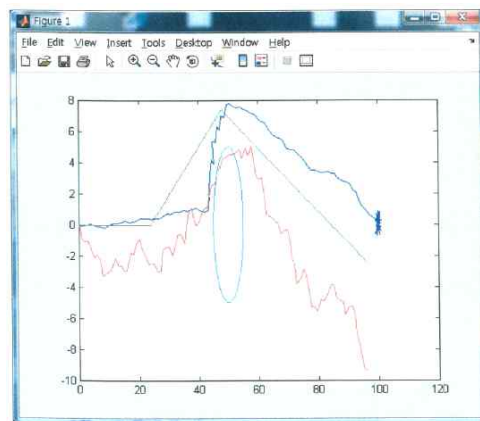
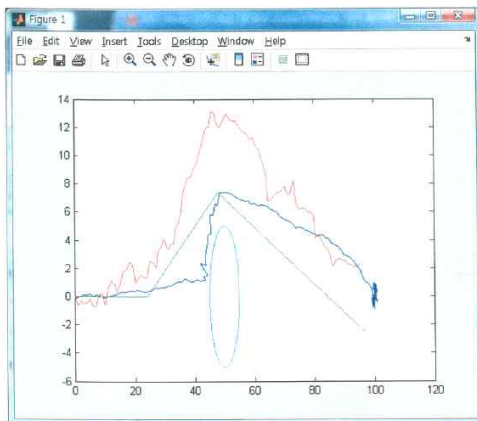
$$1.16 \quad \text{등호성립시 : } (k-1)^2 = 2$$

$$k = 1 - \sqrt{2}$$

$$1.17 \quad \text{As } T \rightarrow \infty, s(z) \rightarrow \sqrt{2} - 1$$

문제 1.16과 일치

1.18



1.19  $\sqrt{3}$  으로수렴

1.20  $|a_{n+2}| \geq |a_{n+1}|^2$   
 $\log|a_n| \geq 2^n \log|a_1|$

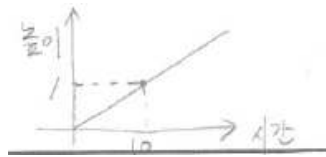
$a_n$ 은 발산한다.  $u_n = a_{n^3} - \frac{3}{2}a_n$ 이라고 하면

$a_{n+1} = 2a_n - a_{n^3} + u_n = \frac{1}{2}a_n$ ,  $a_n = \left(\frac{1}{2}\right)^{n-2}$  수렴한다.

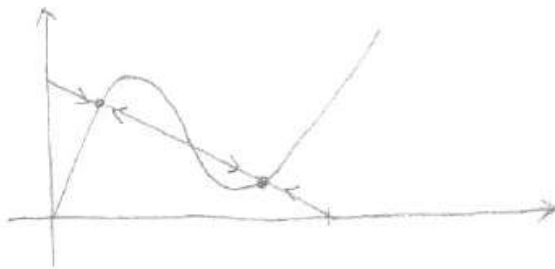
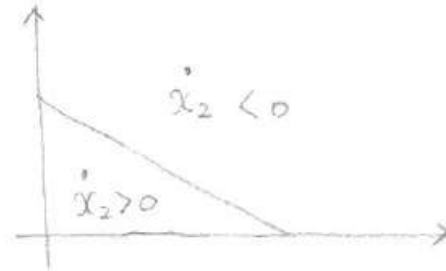
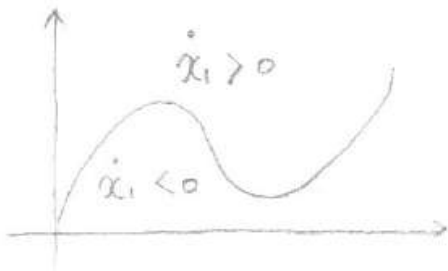
1.21 가속할 때  $v = M\sqrt{\frac{2(x-x_0)}{M}} = \sqrt{2M(x-x_0)}$   
 멈출 때  $v = \sqrt{2N(x_1-x)}$

1.22  $-20 \ell nd$ 마다 한 번씩 물을 채워야 한다.

$\lim_{d \rightarrow 1} \frac{2(1-d)}{-20 \ell nd} = \frac{1}{10}$



1.23 가운데 점  $b$ 는 불안정하고, 양 옆의 점  $a$ 와  $c$ 는 안정하다.  
 스위칭기작이란 시스템의 출력이 점차적으로 변하지 않고,  
 갑자기 변하는 것을 의미한다.  
 그림 우측을 보자 입력이 증가하면 직선이 위로 이동하게 되어,  
 안정점과 불안정점이 만나게 되고 (점  $d$ ) 조금만 더 증가하면  
 안정점이 순간적으로 이동하게 됨을 알 수 있다.  
 반대로 입력이 감소하여 직선이 아래로 내려갈 때도 어느 순간에  
 안정점과 불안정점이 만나고 (점  $e$ ) 같은 원리로 어느 순간에 스위칭 조작이 일어난다.



## Chapter 02 연습문제 답안

2.1 (a)  $x(t) = \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t}$

(b)  $x(t) = \frac{1}{9}t + \frac{2}{27} - \frac{2}{27}e^{3t} + \frac{10}{9}te^{3t}$

(c)  $x(t) = \frac{1}{20}t^5e^{2t}$

(d)  $x(t) = \frac{1}{2} - \frac{1}{2}e^t \cos t + \frac{1}{2}e^t \sin t$

(e)  $x(t) = \cos t$

(f)  $x(t) = \sin t + [1 - \cos(t - \pi)]u_s(t - \pi) - [1 - \cos(t - 2\pi)]u_s(t - \pi)$

2.2 ②

2.3 ①

2.4 ①

2.5 1)  $\frac{6}{(s+2)^4}$

2)  $\frac{1}{2} \left[ \frac{1}{s+1} - \frac{s+1}{(s+1)^2+4} \right]$

3)  $\frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}$

4)  $\frac{s}{s^2+4}e^{-\pi s}$

5)  $\frac{1}{s^2} - \frac{e^{-2s}}{s^2} - 2\frac{e^{-2s}}{s}$

2.6 1)  $e^{-t} - te^{-1}$

2)  $5 - t - 5e^{-t} - 4te^{-t} - \frac{3}{2}t^2e^{-t}$

3)  $\frac{e^{-2s}}{s^2} + \frac{2e^{-2s}}{s}$

4)  $\frac{s}{s^2 + 4}e^{-\pi s}$

5)  $\frac{1}{s^2} - \frac{e^{-2s}}{s^2} - 2\frac{e^{-2s}}{s}$

2.7 ③

2.8  $x(t) = 2e^{-t} + t - 1$

2.9 ②

2.10 ④

2.11 ③

2.12  $\frac{1}{1 - e^{-\pi s}} \frac{1}{s^2 + 1}$

2.13  $\frac{v_{out}}{v_{in}} = \frac{-uR_4}{(R_2 + R_4)R_3sc + (k+1)R_2 + R_3 + R_4}$

2.14 ②

2.15 (a)  $\frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{Rc}}{s + \frac{1}{Rc}}$

(b)  $\frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_1C_2 + R_2C_2)s + 1}$

그림 (a)에서 무한대 부하임피던스를 가정하고 구한 전달함수이다.  
그림 (b)에는 부하단에 유한임피던스를 갖는 회로가 있어 그대로 전달함수를 곱하면 안된다.  
이럴 때는 무한의 부하효과를 내는 *Voltage follower*를 사용하면 된다.  
(부하단에 전류를 흘리지 않아 무한의 부하효과를 낼 수 있다.)

$$2.16 \quad -\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + V(x) \psi(x) = E \psi(x) \quad \text{그리고} \quad i \hbar \frac{df(t)}{dt} = E f(t)$$

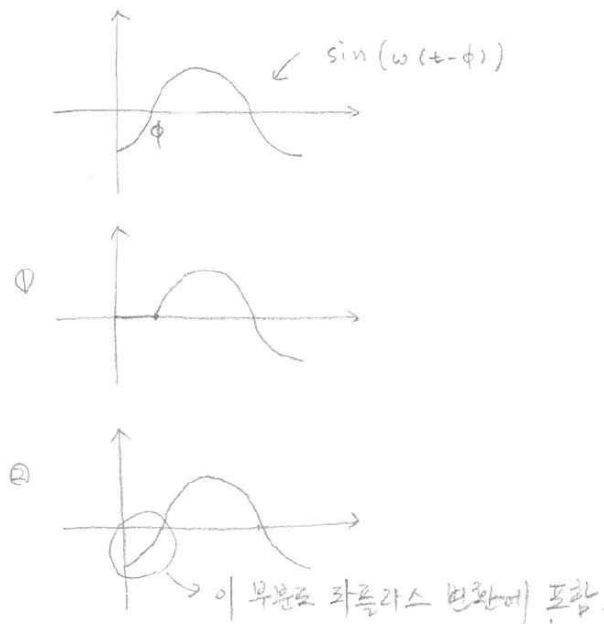
$$2.17 \quad \frac{d}{dt} \begin{bmatrix} soc \\ V \end{bmatrix} = \begin{bmatrix} o & o \\ o & -\frac{1}{Rc} \end{bmatrix} \begin{bmatrix} soc \\ V \end{bmatrix} + \begin{bmatrix} -\frac{1}{c_n} \\ \frac{1}{c} \end{bmatrix} i$$

$$2.18 \quad e^{-\frac{sx^2}{2}}$$

$$2.19 \quad \frac{Y(s)}{\square(s)} = \frac{e^{-2s}}{s^2 + 3s + 5}$$

$$2.20 \quad \frac{d}{dt} \begin{bmatrix} x \\ x' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -w_n^2 & -2\xi w_n \end{bmatrix} \begin{bmatrix} x \\ x' \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

2.21



$$2.22 \quad \frac{d}{dt} f(t) = \frac{1}{2\pi j} \frac{c+j\infty}{c-j\infty} [F(s) - f(0)] e^{st} ds$$

$$L\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0)$$

2.23 a)  $\frac{\sin t}{t}$

b)  $\frac{e^{-t} - e^{3t}}{t}$

c)  $\frac{\sin 2t}{t}$

2.24 a)  $\frac{\pi}{2}$

b)  $\frac{a!b!}{(a+b+1)!}$

c)  $\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \dots$

2.25  $e^{-dt} f_2(t) = -\frac{t}{2\beta^2} \cos \beta t + \frac{1}{2\beta^3} [\sin \beta t]$

$$f_2(t) = e^{2t} \left[ -\frac{t}{2\beta^2} \cos \beta t + \frac{1}{2\beta^3} \sin \beta t \right]$$

$$L^{-1} \left[ \frac{s+1}{[(s-\alpha)^2 + \beta^2]^2} \right] = \frac{df_2(t)}{dt} + f_2(t)$$

2.26  $\frac{d}{dt} \begin{bmatrix} V_o \\ I_L \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{c} \\ -\frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} V_o \\ I_L \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u(t) + \begin{bmatrix} -\frac{1}{c} \\ 0 \end{bmatrix} I_0(t)$

2.27  $m_2$  관련 전달함수 =  $\frac{s(X_1(s)tX_2(s))}{F(s)} = s(B_{11}(s) + B_{21}(s))$

$m_3$  관련 전달함수 =  $\frac{s(X_1(s) + X_3(s))}{F(s)} = s(B_{11}(s) + B_{31}(s))$

2.28  $\frac{abs}{(m_1 + \frac{1}{2}m_2)b^2s^2 + 2kb^2}$

2.29  $\frac{abs}{(m_1 + \frac{1}{2}m_2)b^2s^2 + 2kb^2}$



## Chapter 03 연습문제 답안

3.1 ①

3.2 ①

$$3.3 \quad a) \frac{Y(s)}{\square(s)} = \frac{bs + k}{ms^2 + bs + k}$$

$$b) y = [b \quad k] \begin{bmatrix} g' \\ g \end{bmatrix}$$

$$c) y = [\frac{1}{m} \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$d) \begin{aligned} x' &= Ax + Bu' \rightarrow x' = Ax + ABu \\ y &= cx \rightarrow y = cx + cBu \end{aligned}$$

3.4 ①

3.5 ①

3.6 ①

$$3.7 \quad \frac{G_2 G_3}{1 - G_1 G_2 H_1 + G_2 H_3 + G_2 G_3 H_2}$$

$$3.8 \quad \begin{aligned} \frac{Y}{R} &= \frac{G_1 G_2}{1 - G_1 G_2 H_1 H_2} \\ \frac{Y}{U_1} &= \frac{G_2}{1 - G_1 G_2 H_1 H_2} \\ \frac{Y}{U_2} &= \frac{G_1 G_2 H_1}{1 - G_1 G_2 H_1 H_2} \end{aligned}$$

3.9 32

$$3.10 \quad \begin{aligned} \delta x &= 2x_0 \delta x - 2u_0 \delta u \\ \delta y &= u_0 \delta x + x_0 \delta u \end{aligned}$$

$$3.11 \quad (a) \frac{s+1}{s^2+3s+2}$$

$$(b) \frac{s^2+3s}{s^4+10s^3+6s^2+s+7}$$

$$(c) \frac{e^{-sT}}{s+1}$$

$$3.12 \quad (a) 1 - \frac{7}{3} \frac{s}{s+1} + \frac{3}{2} \frac{s}{s+2} - \frac{1}{6} \frac{s}{s+4}$$

$$(b) \frac{8}{s} - \frac{4s}{s+4} - \frac{2s}{s+5}$$

$$3.13 \quad \frac{\theta(s)}{F(s)} = \frac{\frac{3}{4}L}{\frac{7}{48}mL^2s^2 + \frac{9}{16}KL^2}$$

$$3.14 \quad (a) \begin{bmatrix} 0 \\ F \end{bmatrix}$$

$$(b) \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$(c) \begin{bmatrix} F \\ 0 \end{bmatrix}$$

$$3.15 \quad \frac{V_0}{V_i} = \frac{Z_3Z_1 + Z_2(Z_1 + Z_3 + Z_4)}{Z_2(Z_1 + Z_3 + Z_4) + Z_1Z_3 + Z_1Z_4}$$

$$\frac{V_0}{V_i} = -\frac{Z_4Z_2 - Z_3Z_1}{Z_3(Z_1 + Z_2)}$$

$$3.16 \quad (a) \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

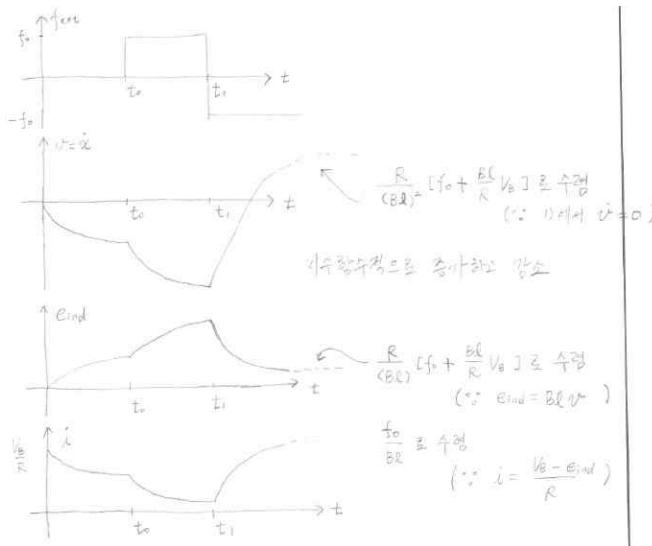
$$(b) \quad v_1 = -R_2C_2 \frac{dv_2}{dt}, v_o = v_2$$

$$3.17 \quad \frac{k_1b_2 + k_1b_1s}{s^2 + a_1s + a_2}$$

$$3.18 \quad \frac{G_1G_2G_3G_4G_5G_8 + G_1G_6G_4G_5G_8 + G_1G_2G_7G_8(1 + G_4H_1)}{1 + [G_4H_2 + G_3G_4G_5H_3 + G_2G_3G_4G_5H_1 + G_6G_4G_5H_1 + G_2G_7H_1 + G_7H_3]} + [G_4H_2G_7H_3]$$

$$3.19 \quad \begin{aligned} f_{ext} - B\ell v &= -mx'' \\ e_{ind} &= B\ell x' = -B\ell v \end{aligned}$$

$$\begin{aligned} \therefore f_{ext} - B\ell \frac{V_\beta - e_{ind}}{R} &= f_{ext} - B\ell \frac{V_\beta - B\ell x'}{R} = -mx'' \\ x' &= v \\ f_{ext} - \frac{B\ell V_\beta}{R} &= -mv' - \frac{(B\ell)^2}{R} v \quad \rightarrow 1) \end{aligned}$$



$$\begin{aligned} V_\beta = 0 \text{ 이면 } 1) \text{에서 } F_{ext}(s) &= -ms V(s) - \frac{(B\ell)^2}{R} V(s) \\ \therefore \frac{V(s)}{F_{ext}(s)} &= -\frac{R}{s + (B\ell)^2} \end{aligned}$$

$$3.20 \quad \begin{aligned} A &\text{ 적분기} \\ B &\text{ 이득} \end{aligned}$$

$$3.21 \quad ①$$

$$3.22 \quad (\text{화살표 방향으로})$$

$$0, \frac{1}{Ls + R_a}, K_i, 0, \frac{1}{Js + B}, K_E$$

$$3.23 \quad \begin{aligned} m_1 x''_1 + b_1 x'_1 + k_1 x_1 + b_2 (x'_1 - x'_2) + k_2 (x_1 - x_2) &= p(t) \\ m_2 x''_2 + b_2 (x'_2 - x'_1) + k_2 (x_2 - x_1) &= 0 \end{aligned}$$

$$3.24 \quad \frac{Z(s)}{Y(s)} = \frac{-ms^2}{ms^2 + bs + k}$$

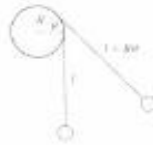
$$3.25 \quad \text{풀이 준비 중}$$

### 3.26

1. The figure below shows a simple pendulum system in which a cord is wrapped around a fixed cylinder. The motion of the system that results is described by the differential equation:

$$(l + R\theta)\ddot{\theta} + g \sin(\theta) + R\dot{\theta}^2 = 0,$$

where  $l$  is the length of the cord in the vertical (down) position and  $R$  is the radius of the cylinder.



$$\begin{aligned} \frac{d}{dt} \left[ m(l + R\theta) \dot{\theta} \right] &= -mg \sin(\theta) (l + R\theta) \\ m(l + R\theta) \ddot{\theta} &= -mg \sin(\theta) (l + R\theta) \\ \ddot{\theta} &= \frac{-g \sin(\theta)}{l + R\theta} \end{aligned}$$

- Write the state space equations for this system.
- Linearize the equation around the point  $\theta = 0$ ,  $\dot{\theta} = 0$ , and show that for small values of  $\theta$  the system equation reduces to an equation for a simple pendulum, that is,  $\ddot{\theta} + (g/l)\theta = 0$ . (25 pts)
- Let  $x_1(t) = \theta$ ,  $x_2(t) = \dot{\theta}$  and  $y(t) = \theta$ . Substituting these into the differential equation gives:

$$\begin{aligned} (l + Rx_1(t))\ddot{x}_2(t) + g \sin(x_1(t)) + Rx_2(t)^2 &= 0 \\ \Rightarrow \dot{x}_2(t) &= \frac{-g \sin(x_1(t)) - Rx_2(t)^2}{l + Rx_1(t)} \end{aligned}$$

the rate of change of the second state variable with respect to the present values of the state variables. We can determine the rate of change of the first state variable directly from its definition:

$$\dot{x}_1(t) = \dot{\theta} = x_2(t).$$

Likewise, from its definition,  $y(t) = \theta = x_1(t)$ .

The state space equations for this system are then:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= \frac{-g \sin(x_1(t)) - Rx_2(t)^2}{l + Rx_1(t)}, \quad \text{and} \\ y(t) &= x_1(t). \end{aligned}$$

- b. To linearize a state space model with two state variables  $x_1(t)$  and  $x_2(t)$  and one output variable  $y(t)$ , i.e.,

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} f_1(\mathbf{x}(t)) \\ f_2(\mathbf{x}(t)) \end{bmatrix},$$

$$y(t) = g(\mathbf{x}(t)),$$

around an operating point  $(x_{1Q}, x_{2Q}, y_Q)$ , we define three new variables,  $\Delta x_1(t) = x_1(t) - x_{1Q}$ ,  $\Delta x_2(t) = x_2(t) - x_{2Q}$ , and  $\Delta y(t) = y(t) - y_Q$ , and use the following 1<sup>st</sup>-order Taylor series approximations:

$$\Delta \dot{\mathbf{x}}(t) = \begin{bmatrix} \Delta \dot{x}_1(t) \\ \Delta \dot{x}_2(t) \end{bmatrix} \approx \begin{bmatrix} \left. \frac{\partial f_1}{\partial x_1} \right|_{\mathbf{x}=\mathbf{x}_Q} & \left. \frac{\partial f_1}{\partial x_2} \right|_{\mathbf{x}=\mathbf{x}_Q} \\ \left. \frac{\partial f_2}{\partial x_1} \right|_{\mathbf{x}=\mathbf{x}_Q} & \left. \frac{\partial f_2}{\partial x_2} \right|_{\mathbf{x}=\mathbf{x}_Q} \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix},$$

$$\Delta y(t) \approx \begin{bmatrix} \left. \frac{\partial g}{\partial x_1} \right|_{\mathbf{x}=\mathbf{x}_Q} & \left. \frac{\partial g}{\partial x_2} \right|_{\mathbf{x}=\mathbf{x}_Q} \end{bmatrix} \begin{bmatrix} \Delta x_1(t) \\ \Delta x_2(t) \end{bmatrix}.$$

For the state space equations derived in part a, these yield:

$$\Delta \dot{x}_1(t) = \Delta x_2(t),$$

$$\Delta \dot{x}_2(t) = \left[ -\frac{g \cos(x_{1Q})}{(l + R x_{1Q})} + \frac{R g \sin(x_{1Q})}{(l + R x_{1Q})^2} \right] \Delta x_1(t) - \frac{2 R x_{2Q}}{(l + R x_{1Q})} \Delta x_2(t), \text{ and}$$

$$\Delta y(t) = \Delta x_1(t).$$

The first and last of these equations are independent of the operating point. Evaluating the second equation for the given operating point  $\theta = 0$ ,  $\dot{\theta} = 0 \Rightarrow x_{1Q} = x_{2Q} = y_Q = 0$ , gives:

$$\Delta \dot{x}_2(t) = \left[ -\frac{g \cos(0)}{(l + R \cdot 0)} + \frac{R g \sin(0)}{(l + R \cdot 0)^2} \right] \Delta x_1(t) - \frac{2 R \cdot 0}{(l + R \cdot 0)} \Delta x_2(t)$$

$$= -\frac{g}{l} \Delta x_1(t)$$

Substituting our definitions of  $\Delta x_1(t)$  and  $\Delta x_2(t)$  into this equation produces:

$$\ddot{\theta} = -\frac{g}{l} \theta \Rightarrow \ddot{\theta} + (g/l)\theta = 0,$$

the equation for a simple pendulum.

[The equations for  $\Delta \dot{x}_1(t)$  and  $\Delta y(t)$  follow directly from the definitions and provide no further information about the behaviour of the system.]

### 3.27 풀이 준비 중

$$3.28 \quad E_0 = \sqrt{\frac{k x_1 h}{L_0}} \left(1 + \frac{x_0}{2h}\right) R$$

### 3.29 풀이 준비 중

$$3.30 \quad G_{11}(s) = \frac{G_1}{1 - G_1 G_2 G_3 G_4}$$

$$G_{21}(s) = \frac{-G_1 G_3 G_4}{1 - G_1 G_2 G_3 G_4}$$

$$G_{22}(s) = \frac{G_4}{1 - G_1 G_2 G_3 G_4}$$

$$G_{12}(s) = \frac{-G_1 G_2 G_4}{1 - G_1 G_2 G_3 G_4}$$

$$3.31 \quad \therefore dW_e = f \cdot dx$$

$$\therefore \frac{\partial W_e(x_1 e)}{\partial x} = f$$

$$3.32 \quad \frac{V_0(s)}{T(s)} = \frac{Br^2}{2Js}$$

$$3.33 \quad \therefore \begin{bmatrix} V \\ I \end{bmatrix} = \exp \left( \begin{bmatrix} 0 & -(p+jwL) \\ -(G+jwc) & 0 \end{bmatrix} Z \right) \begin{bmatrix} V_{(0)} \\ I_{(0)} \end{bmatrix}$$

$$V = V_0^+ e^{-rZ} + V_0^- e^{rZ}$$

$$I = I_0^Z e^{-rZ} + I_o^- e^{rZ}$$

$e^{-j\beta Z}$  항에 의해 어떤 정해진 시각  $t$ 에서 보면 공간  $Z$ 에 대하여 진동한다.  
 $e^{-dZ}$ 에 의해  $V$ 와  $I$ 는 감쇠한다.

$$3.34 \quad \frac{\sinh(\sqrt{s} x)}{\sinh(\sqrt{s})} = \sum_{n=0}^{\infty} [e^{-(2n+1-x)\sqrt{s}} - e^{-(2n+1+x)\sqrt{s}}]$$

3.35 풀이 준비 중

## Chapter 04 연습문제 답안

4.1 ③

4.2 ①

4.3 ①

4.4 ①

4.5 ④

4.6 ④

$$4.7 \quad \frac{\sqcup(s)}{R(s)} = \frac{C(s)}{1 + C(s)G(s)}$$

$$4.8 \quad \frac{\sqcup_i(s) - \sqcup_r(s)}{\sqcup_i(s)} = \frac{\sqcup_i(s) + G(s)C(s)\sqcup_i(s)}{\sqcup_i(s)} = 1 + C(s)G(s)$$

4.9 ④

$$4.10 \quad \therefore |1 + (C_{(jw)}G_{(jw)})^{-1}| > |\Delta_{(jw)}|$$

$|\Delta_{(jw)}| = |\ell_{0(jw)}|$  이므로

$$\left| \frac{1}{T_{(jw)}} \right| = |1 + (C_{(jw)}G_{(jw)})^{-1}| > |\ell_{0(jw)}| \text{ 만족하면 폐로 시스템을 안정하라.}$$

$CAACSD$ 를 사용하면 안정함을 확인할 수 있다.

4.11 (a) 안정 (b) 불안정 (c) 불안정

4.12 (a) 0개 (b) 2개 (c) 2개

4.13 (a)  $K > -2$  (b)  $0 < K < 784$  (c)  $-2 > K < 4$

4.14 ①

4.15  $0 < k < 3.8854$

4.16 ①

4.17  $v_2 = \frac{R_2 R_4 C_2}{R_3} v'_1$

$v_1 = -R_1 C_1 v'_2$

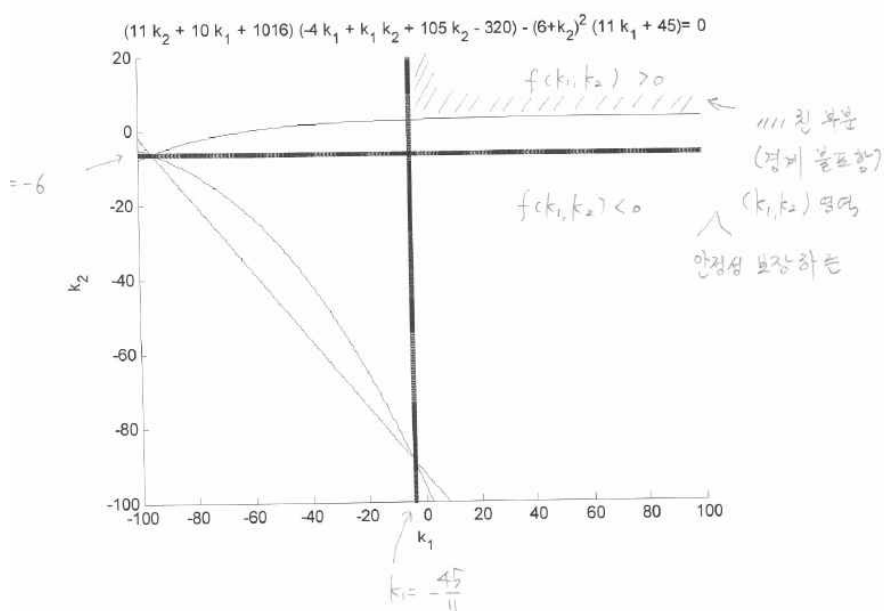
$$\begin{bmatrix} v'_1 \\ v'_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{R_3}{R_2 R_4 C_2} \\ -\frac{1}{R_1 C_1} & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

시스템 행렬의 고유치가 허수축에 있으므로 진동 (불안정)

4.18 ①

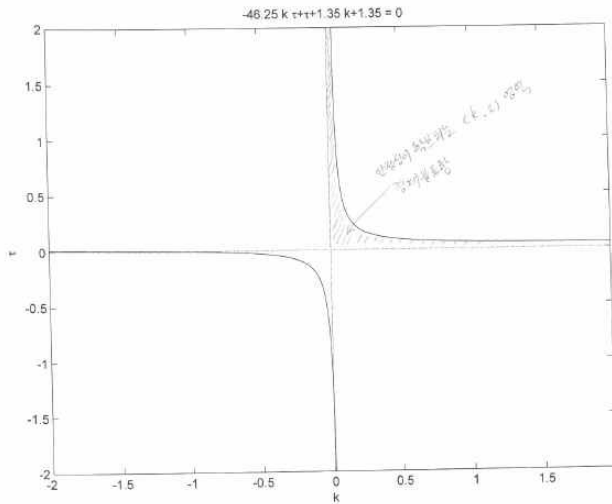
4.19  $k > -1 + \frac{\sqrt{17}}{2}$

4.20  $k_1 > -\frac{45}{11}, k_2 > -6$



4.21





4.22  $\|e\|_{\infty} = 1$  ( $t=0$  에서 발생)  $e(t) = e^{-\frac{1}{T}t}$

4.23  $\|e\|_2 = \left| \frac{1}{2\pi} \right| \pi \frac{1+4\xi^2}{2W_n \sin\theta} = \sqrt{\frac{1+4\xi^2}{4W_n \sin\theta}} = \sqrt{\frac{1+4\xi^2}{4\xi W_n}}$

4.24 풀이 준비 중

4.25  $\left| 1 + \frac{C_{(jw)}\Delta_{a(jw)}}{1 + G_{(jw)}C_{(jw)}} \right| > 0$  을 만족하면 된다.

$$\left| 1 + \frac{C_{(jw)}\Delta_{a(jw)}}{1 + G_{(jw)}C_{(jw)}} \right| > 1 - \left| \frac{C_{(jw)}\Delta_{a(jw)}}{1 + G_{(jw)}C_{(jw)}} \right|$$

$$\geq 1 - \left\| 1 + \frac{C_{(jw)}\Delta_{a(jw)}}{1 + G_{(jw)}C_{(jw)}} \right\|_{\infty} > 0$$

4.26  $1 + G_{\Delta(s)}C(s) = 1 + G(s)C(s)(1 + \Delta_m(s))$

$$= [1 + G(s)C(s)] \left[ 1 + \frac{G(s)C(s)}{1 + G(s)C(s)} \Delta_m(s) \right]$$

$$\left| 1 + \frac{G(s)C(s)}{1 + G(s)C(s)} \Delta_m(s) \right| > 1 - \left\| \frac{G(jw)C(jw)}{1 + G(jw)C(jw)} W_m(jw) \right\|_{\infty}$$

$$> 0$$

4.27 풀이 준비 중

4.28 풀이 준비 중

4.29 풀이 준비 중

4.30  $k = \sqrt{2}$

## Chapter 05 연습문제 답안

$$5.1 \quad = \frac{1}{s^2}(1 - e^{-s})$$

$$5.2 \quad \textcircled{4}$$

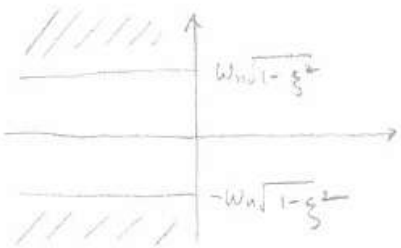
$$5.3 \quad \textcircled{2}$$

$$5.4 \quad \textcircled{4}$$

$$5.5 \quad 4.0746 \times 10^{-4}$$

$$5.6 \quad k = \frac{100}{9}$$

$$5.7$$



$$5.8 \quad \frac{Y(s)}{R(s)} = \frac{10(3+3s)}{s(s^2+s+20)+10(3+3s)}$$

$$\frac{Y(s)}{W(s)} = \frac{10s}{s(s^2+s+20)+10(3+3s)}$$

$Y$ 에 대해서 1형 시스템  
 $W$ 에 대해서 1형 시스템

$$5.9 \quad \frac{1}{K_a} = \frac{1}{2} \left[ \sum_{i=1}^n \frac{1}{Z_i^2} - \sum_{i=1}^n \frac{1}{P_i^2} \right]$$

5.10 1형 시스템  $\frac{1}{kv} = \int_0^{\infty} h_{(t),t} dt$

2형 시스템  $\int_0^{\infty} h_{(2),dZ} = 1$  ,  $\int_0^{\infty} zh_{(z)} dz = 0$

5.11 계단입력

$$Y_{(s)} = G_{(s)} \frac{1}{s}$$

$$y_{ss}(t) = \lim_{s \rightarrow 0} s G_{(s)} \frac{1}{s} = G_{(0)}$$

햄프 입력

$$Y_{(s)} = G_{(s)} \frac{1}{s^2} = \frac{G'_{(0)}}{s} + \frac{G_{(0)}}{s^2} + \dots$$

$$y_{ss}(t) = G'_{(0)} + G_{(0)}t$$

포괄선 입력

$$Y_{(s)} = G_{(s)} \frac{1}{s^3} = \frac{G''_{(0)}}{2s} + \frac{G'_{(0)}}{s^2} + \frac{G_{(0)}}{s^3} + \dots$$

$$y_{ss}(t) = \frac{1}{2} G''_{(0)} + G'_{(0)}t + G_{(0)} \frac{t^2}{2}$$

5.12 풀이 준비 중

5.13  $\int_0^{tp} h_{(t)} dt = 1 + Mp$

5.14  $\lim_{s \rightarrow 0} \left[ 1 - \frac{G_{(s)}}{1 + G_{(s)}} \right] \frac{1}{s}$

$$= \lim_{s \rightarrow 0} \frac{1}{1 + G_{(s)}} \frac{1}{s} = \lim_{s \rightarrow 0} \frac{1}{G_{(s)} \cdot s} = \frac{1}{kw}$$

5.15 풀이 준비 중

5.16 정상상태 오차 = 0 되도록 조건을 구한다.

$$k_2(k_1 - 1) = 1$$

$$k_1 = 1 + \frac{1}{k_2}$$

$$\frac{k_2 + 1}{2} = \frac{1}{k_2 + 1} \text{ 에서 최소 발생}$$

$$k_2 = \sqrt{2} - 1$$

$$5.17 \quad \alpha = 175$$

$$\beta = \frac{200^2 - 25 \times 175}{50} = 712.5$$

$$5.18 \quad \begin{array}{cc} \frac{Y_1}{W_1} & 1 \text{형} & \frac{Y_2}{W_1} & 1 \text{형} \\ \frac{Y_1}{W_2} & 1 \text{형} & \frac{Y_2}{W_2} & 0 \text{형} \end{array}$$

## Chapter 06 연습문제 답안

6.1 ④

6.2 ④

6.3 ①

6.4  $\alpha > -\frac{1}{2}$  이어야 안정화

6.5 ②

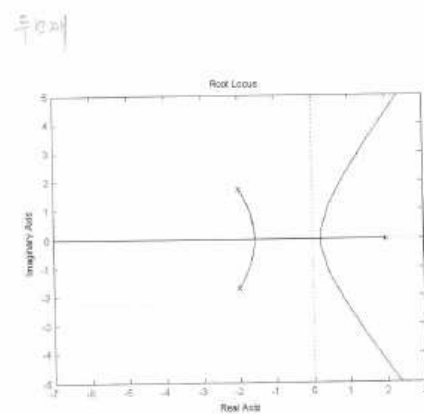
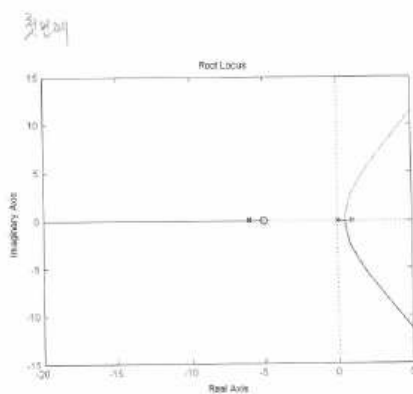
6.6 ①

6.7 ④

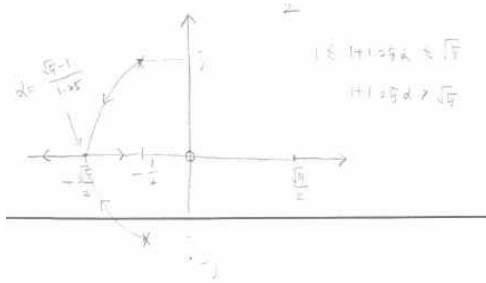
6.8 ④

6.9 ④

6.10



## 6.11

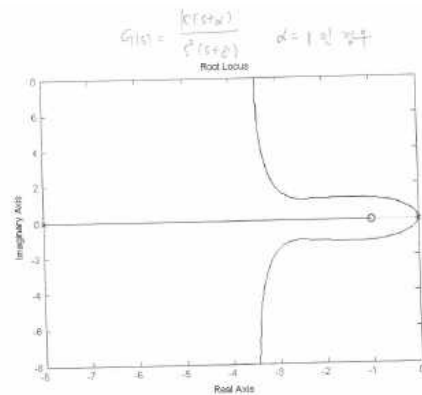
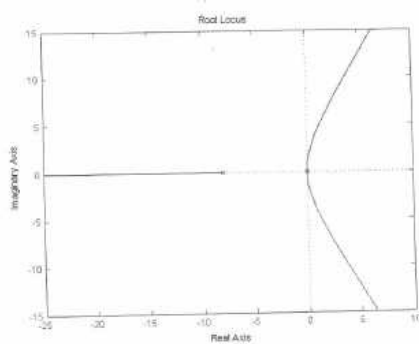


$$6.12 \quad G(s) = \frac{N(s)}{D(s)} = \frac{(s + Z_1) \cdots (s + Z_n)}{(s + p_1) \cdots (s + p_n)} - \frac{d}{ds} \left( \frac{(s + p_1) \cdots (s + p_n)}{(s + z_1) \cdots (s + z_n)} \right) = 0$$

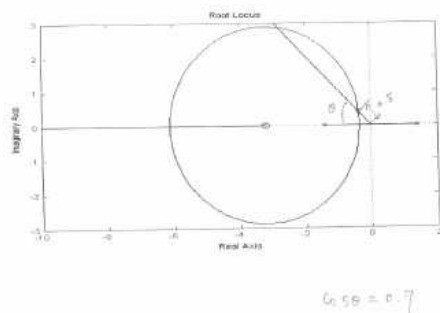
$$\Rightarrow \sum_{i=1}^n \frac{1}{\alpha + \pi} \left[ \frac{D(\alpha)}{N(\alpha)} \right] - \sum_{i=1}^m \frac{1}{\alpha + Zi} \left[ \frac{D(\alpha)}{N(\alpha)} \right] = 0$$

$$\left[ \sum_{i=1}^n \frac{1}{\alpha + \pi} - \sum_{i=1}^m \frac{1}{\alpha + Zi} \right] = 0$$

6.13  $k > 0$ 에 대하여 우반평면에 2개의 극점이 항상 존재.  $\therefore$  불안정함  
 $0 < \alpha < 8$  이면 항상  $8k - k\alpha > 0 \therefore$  안정함

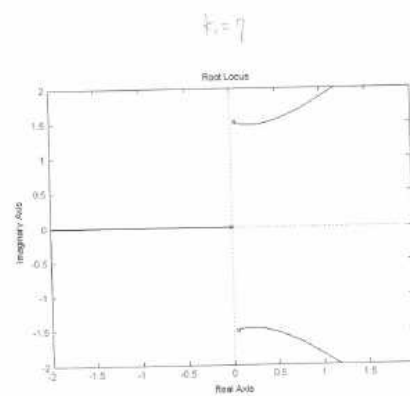
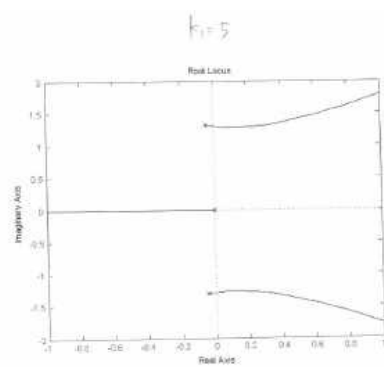
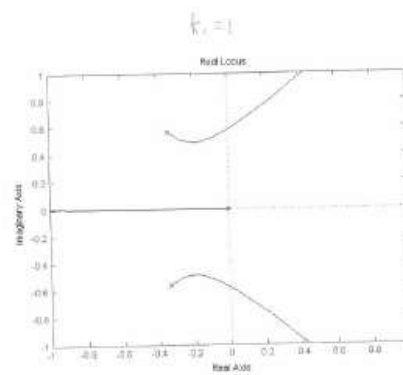
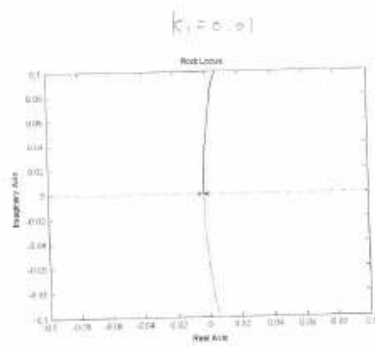
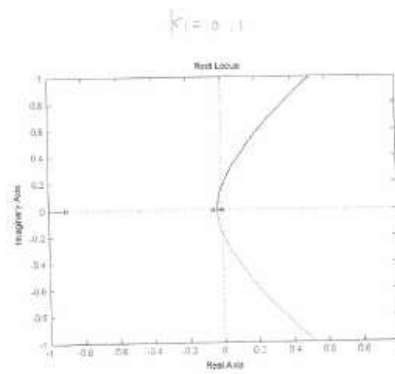
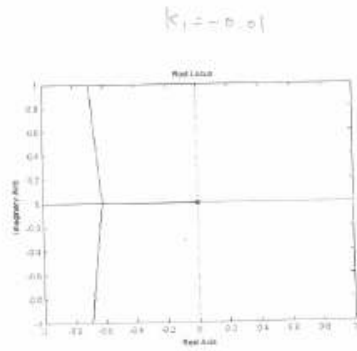


$$6.14 \quad \alpha = 2.25, \quad \beta = \frac{0.7}{2.25}$$



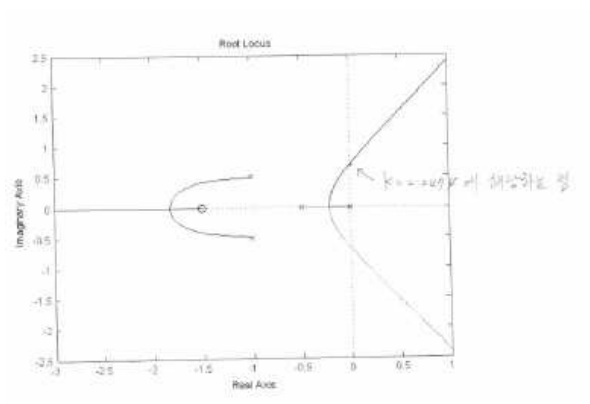
6.15  $k = 0.3427$

6.16  $k_1 < 6$   
 $k_2 < \frac{6 - k_1}{9}$

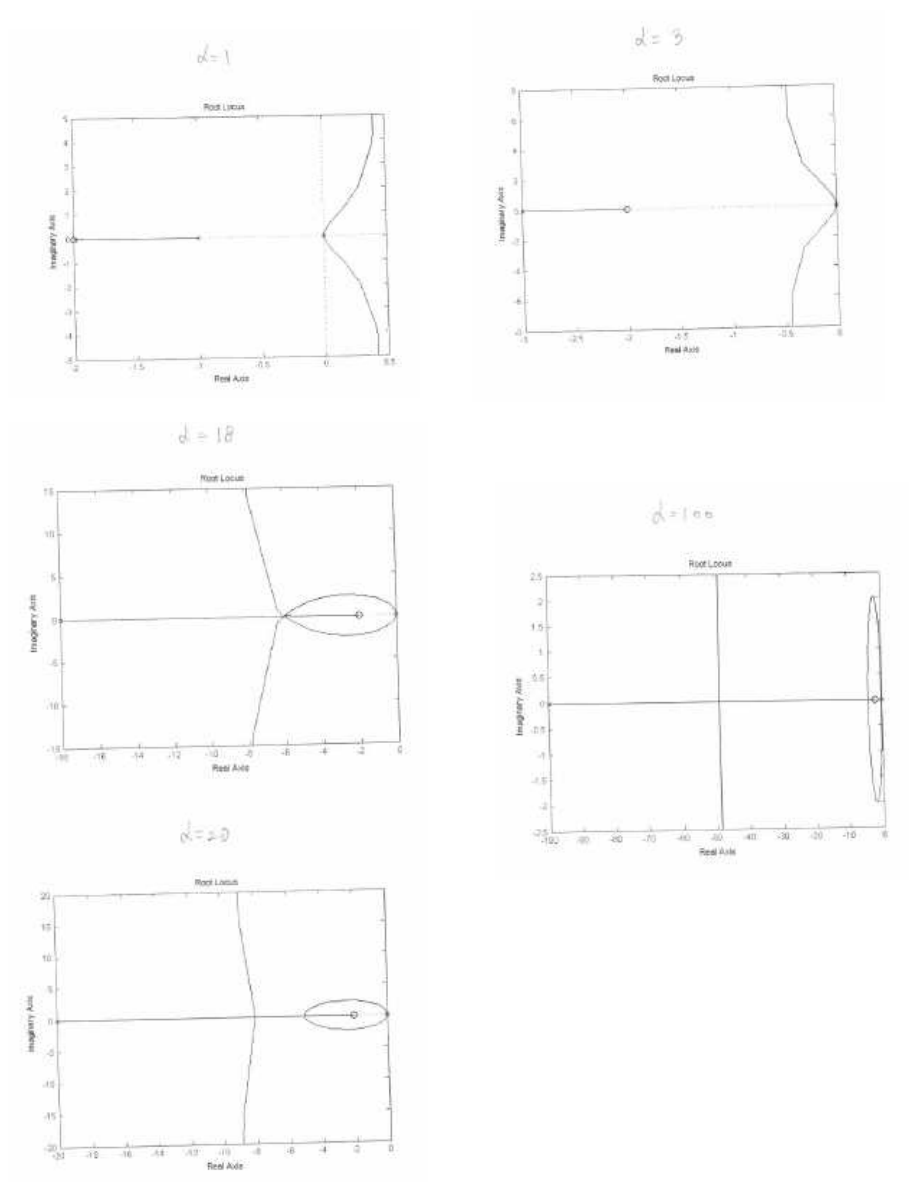


6.17  $0 < k < -10 + \sqrt[5]{6} (= 2.2474)$





6.18



## Chapter 07 연습문제 답안

7.1 ①

7.2 ①

7.3 ④

7.4 ①

7.5 ①

7.6 ①

7.7 ①

7.8 ①

7.9 ①

7.10 ②

7.11 ②

7.12 ④

7.13  $\frac{1}{11}$

7.14 ①

7.15 ①

7.16 앞정보상기  $\phi = \sin^{-1} \frac{1-\alpha}{1+\alpha}$   
뒤집보상기  $\phi = \sin^{-1} \frac{\beta-1}{1+\beta}$

7.17 ③

7.18 ①

7.19  $\alpha = \frac{2}{\sqrt[4]{8}}$

7.20 ④

7.21 a)  $k > 3$   
b)  $0 < k < \frac{2}{T}$   
c)  $k > \frac{1}{2T_3}$

7.22 풀이 준비 중

7.23 a)  $0 < k < \frac{\pi}{8}$   
b)  $0 < k < \frac{\sqrt[2]{2}}{\pi^2}$

7.24 Lemma :  $\int_{-\infty}^{\infty} \lim \left| \frac{jw-\alpha}{jw-\beta} \right|^2 dw = 2\pi(|Re_{(\alpha)}| - |Re_{(\beta)}|)$   
 $\sum(s) = \frac{1}{1+G(s)} = \frac{\pi(s-\pi)}{\pi(s-pn) + \pi(s-zn)} = \frac{\pi(s-\pi)}{\pi(s-ri)}$

최고 차형과다음차 계수가 같다.  $\sum \pi = \sum ri$

$$\begin{aligned} \int_{-\infty}^{\infty} \lim |S_{(jw)}| dw &= \pi \sum_{i=1}^n (|Re(\pi)| - |Re(ri)|) \\ &= \pi \left[ - \sum_{i=m+1}^n Re(\pi) + \sum_{i=1}^M Re(\pi) + \sum_{i=1}^n Re(ri) \right] \\ &= \pi 2 \sum_{i=1}^M Re(\pi) \\ &\text{대칭이용} \\ \int_0^{\infty} \lim |S_{(jw)}| dw &= \pi \sum_{i=1}^M Re(\pi) \end{aligned}$$

7.25 풀이 준비 중

7.26 풀이 준비 중

7.27 풀이 준비 중

$$\begin{aligned}
 7.28 \quad \phi &= \pi + \left[ -1 - \tan^{-1} \frac{2}{td} \right] \\
 &= \pi - 1 - \tan^{-1} \frac{2}{td} > \pi - 1 - \frac{\pi}{2} \\
 &= \frac{\pi}{2} - 1 = 32.7042^\circ
 \end{aligned}$$

$$\begin{aligned}
 7.29 \quad \pi + \tan^{-1} 3 - \frac{\pi}{2} - 0.9 - \tan^{-1} \frac{0.9}{L} 2 \\
 = \pi + \tan^{-1} 3 - \frac{\pi}{2} - 0.9 - \tan^{-1} 3 \\
 = \frac{\pi}{2} - 0.9 = 38.43^\circ
 \end{aligned}$$

$$7.30 \quad e^{-2\phi}$$

## Chapter 08 연습문제 답안

8.1 ①

8.2 ①

8.3 ①

8.4 ①

8.5 1)  $s^2 + 3s + 2$

2)  $s^2 + 3s + 1.5^2 - 0.25 = 2$

3)  $s^2 + 3s + 2$

4)  $s^2 + 3s + 1$

8.6 ②

$$8.7 \quad \frac{e^{At}}{e} = \begin{bmatrix} 1 & 0 & 0 \\ t & 1 & 0 \\ \frac{t^2}{2} & t & 1 \end{bmatrix}$$

8.8 풀이 준비 중

$$8.9 \quad \alpha\beta wd = w_n^2, \quad \alpha\beta = \frac{w_n}{\sqrt{1-\xi^2}}$$

$$8.10 \quad k_1 = -\frac{1}{2}, \quad k_2 = 0$$

$$v = -\left[-\frac{1}{2} \quad 0\right]x + r \quad (r \text{은 기준 신호})$$

$$8.11 \quad b^2 \neq ab - 2$$

8.12  $[B \ AB] = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \rightarrow \det = 0$  가 제어하지 않는다.

$$\begin{bmatrix} c \\ cA \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix} \rightarrow \det \neq 0 \text{ 가 관측하다.}$$

$$v = -kx + r = -[k_1 \ k_2]x + r$$

$$[B(A - Bk)B] = \begin{bmatrix} 1 & -k_1 & 1 - k_2 \\ 2 & -2 - 2k_1 & 3 - 2k_2 \end{bmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & -k_1 - 2k_2 + 2 \\ 2 & -2k_1 - 4k_2 + 4 \end{bmatrix} \rightarrow \det = 0$$

어떤 제어기로도 가제어성을 획득하지 못한다.

$$\begin{bmatrix} c \\ c(A-B^k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -3k_1-2 & -3k_2+4 \end{bmatrix} \rightarrow \det = -3k_2+4+3k_1+2$$

$k_1 - k_2 + 2 = 0$ 을 만족하는 제어기를 사용할 시 가관측성을 잃어버린다.

8.13  $\alpha = 1, 2, 4$

제어기 표준형에  $\alpha = 1, 2, 4$ 대입 : 가제어성은 있으나, 가관측성이 없는 상태 방정식

가관측성 표준형에  $\alpha = 1, 2, 4$ 대입 : 가고나측성은 있으나, 가제어성은 없는 상태 방정식

$$\begin{aligned} 8.14 \quad k_1 &= \frac{8}{5} \\ k_2 &= \frac{37}{5} \\ k_3 &= \frac{74}{5} \end{aligned}$$

8.15  $y_{(t)} = [0 \ 1] x_{(t)}$

8.16  $k_1 = 99, k_2 = 20$   
 $\ell_1 = 20, \ell_2 = 99$

8.17

## 8.18 풀이 준비 중

$$8.19 \quad w_i^T A = \lambda_i w_i^T \quad i = 1, 2, 3, \dots, n \quad \leftrightarrow \text{제어가능}$$

$$w_i^T B \neq 0$$

제어가능  $\rightarrow \text{rank}[\lambda I - A : B] = \text{제어가능 안함}$

$\text{rank}[\lambda I - A : B] \neq n \exists \lambda \in c \rightarrow \text{제어가능 안함}$

$\text{rank}[\lambda I - A : B] \neq n$ 이면 0 아닌  $\alpha$ 가 존재하여 다음을 만족한다.

$$\alpha^T [\lambda I - A : B] = 0$$

$$\lambda \alpha^T = \alpha^T A, \alpha^T B = 0 \quad \alpha \neq 0 \quad \text{제어가능하지 않음.}$$

8.20 풀이 준비 중

8.21 풀이 준비 중

8.22 풀이 준비 중

$$8.23 \quad x' = \begin{bmatrix} 0 & 1 & 0 \\ -2 & 0 & C \\ 0 & 0 & A \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ B \end{bmatrix} \mu$$

8.24 풀이 준비 중

$$8.25 \quad \text{제어기} : C_{(s)} \\ (A) : D_{(s)}$$

$$8.26 \quad Z_{(t)} = x_{(t)} + \int_{t,h}^t e^{A(t-\theta-h)} B_1 u_{(\alpha)} d\alpha \quad \rightarrow 1)$$

$$z' = x'_{(t)} + A[z_{(t)} - x_{(t)}] + e^{-Ah} B_{1u(t)} - B_{1u(t-h)}$$

$$= A x_{(t)} + B_{1u(t-h)} + A z_{(t)} - A x_{(t)} + e^{-Ah} B_{1u(t)} - B_{1-u(t-h)} + B \cdot u(t)$$

$$= A z_{(t)} + (B_0 + e^{-Ah} B_1) u_{(t)}$$

$$z_{(t)} \rightarrow 0$$

$$u_{(t)} \rightarrow 0$$

$$x_{(t)} \rightarrow 0 \text{ 이면 1)에 의해}$$

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} B_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} h = 1$$

$$\Rightarrow Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = 1 \quad LQR \quad v_{(t)} = -[-2.3937 \quad 5.2530] Z_{(t)}$$