

8.1 연습문제

$$1. (a) \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1/3 & -3 & 1 \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & -1 & 3 \\ 0 & 0 & 8 \end{bmatrix}$$

$$2. (a) \mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & 1 & 0 \\ -5 & 6 & -2 & 1 \end{bmatrix}, \quad \mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$3. (a) \mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 \\ -4 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} 25 & 0 & 0 & 0 & 1 \\ 0 & 27 & 4 & 3 & 2 \\ 0 & 0 & 50 & -6 & -4 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 \end{bmatrix}$$

$$4. (b) \mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$5. (a) \mathbf{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -x/b & 1 & 0 \\ -w/a & (xy)/(bc) & -y/c & 1 \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} a & 0 & 0 & z \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & d - (wz)/a \end{bmatrix}$$

$$(b) \mathbf{L}' = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & x & c & 0 \\ 0 & 0 & y & d - (wz)/a \end{bmatrix},$$

$$\mathbf{U}' = \begin{bmatrix} 1 & 0 & 0 & z/a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$6. \text{ (a) } \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1/4 & 1 & 0 & 0 \\ -1/4 & -1/15 & 1 & 0 \\ 0 & -4/15 & -2/7 & 1 \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} 4 & -1 & -1 & 0 \\ 0 & 15/4 & -1/4 & -1 \\ 0 & 0 & 56/15 & -16/15 \\ 0 & 0 & 0 & 24/7 \end{bmatrix}$$

$$\text{(b) } \mathbf{D} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 15/4 & 0 & 0 \\ 0 & 0 & 56/15 & 0 \\ 0 & 0 & 0 & 24/7 \end{bmatrix},$$

$$\mathbf{U}' = \begin{bmatrix} 1 & -1/4 & -1/4 & 0 \\ 0 & 1 & -1/15 & -4/15 \\ 0 & 0 & 1 & -2/7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{(c) } \mathbf{L}' = \begin{bmatrix} 4 & 0 & 0 & 0 \\ -1 & 15/4 & 0 & 0 \\ -1 & -1/4 & 56/15 & 0 \\ 0 & -1 & -16/15 & 24/7 \end{bmatrix}$$

$$\text{(d) } \mathbf{L}'' = \begin{bmatrix} 2 & 0 & 0 & 0 \\ -1/2 & (1/2)\sqrt{15} & 0 & 0 \\ -1/2 & -1/(2\sqrt{15}) & 2\sqrt{14/15} & 0 \\ 0 & -2/(\sqrt{15}) & -(4/7)\sqrt{14/15} & 2\sqrt{6/7} \end{bmatrix}$$

(e) 192

$$8. \quad \mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -8 \end{bmatrix},$$

$$\mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & -1 & 1 & 1 \end{bmatrix}$$

$$9. \text{ (a) } \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & -1 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$

$$\mathbf{U}' = \begin{bmatrix} 1 & -1/2 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{(b) } \mathbf{x} = [-1, 2, 1]^T$$

$$10. \text{ (a) } \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 3 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$\mathbf{U}' = \begin{bmatrix} 1 & -1/2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{(b) } \mathbf{x} = [-1, 1, 1]^T$$

$$12. \quad \mathbf{A}^{-1} = \frac{1}{15} \begin{bmatrix} 11 & -5 & -7 \\ -13 & 10 & 11 \\ -8 & 5 & 1 \end{bmatrix}$$

$$14. \text{ (a) } \begin{bmatrix} \ell_{11} & \ell_{11}u_{12} & 0 & 0 \\ \ell_{21} & \ell_{21}u_{12} + \ell_{22} & \ell_{22}u_{23} & 0 \\ 0 & \ell_{32} & \ell_{32}u_{23} + \ell_{33} & \ell_{33}u_{34} \\ 0 & 0 & \ell_{43} & \ell_{43}u_{34} + \ell_{44} \end{bmatrix}$$

$$16. \text{ (a) } \chi^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\text{(b) } \chi^{-1} = \begin{bmatrix} 0 & -1 & -1 & 1 \\ -1 & 0 & -1 & 1 \\ -1 & -1 & 0 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

8.1 컴퓨터 연습문제

1. (d) $p_5(A) = \begin{bmatrix} 536 & -668 & 458 & -186 \\ -668 & 994 & -854 & 458 \\ 458 & -854 & 994 & -668 \\ -186 & 458 & -668 & 536 \end{bmatrix}$

8.2 연습문제

1. 그렇다.

3. 기저 벡터 사상: $(1, 0) \rightarrow (\cos\theta, \sin\theta)$
각 θ 를 통해 반시계방향으로 회전시킨다.

7. 고윳값: (a) $9.8393, 3.0804 \pm 1.3763i$
(c) $0.4660 - 1.4971i, 2.6112 + 0.3313i, -0.0773 + 2.1658i$

9. (c)

11. 고윳값/고유벡터: 1, $(-1, -1, 0, 0)$; 2, $(0, 0, -1, 1)$; 5, $(-1, 1, 2, 2)$; 10, $(2, 2, 1, 1)$

13. 참

15. 참

8.2 컴퓨터 연습문제

1. (a) 고윳값: $-2 \pm \sqrt{23} = -6.79583, 2.79583$
고유벡터: $[1, 9 - (3 \pm \sqrt{23})/7]^T$ 또는 $[1, -1.11369]^T$ 그리고 $[1, 0.25655]^T$
(d) $n = 5$: 고윳값: $0.26795, 1, 2, 3, 3.73205$

7. 고윳값/고유벡터: (a) $5.2426, (0.6656, 0.7463)$; $-3.2426, (-0.3051, 0.9523)$
(b) $3.9893 \pm 5.5601i, (0.7267, -0.0680 \pm 0.4533i,$
 $-0.3395 \mp 0.3829i); 0.0214, (0.7916, 0.5137, 0.3308)$

11. 고윳값/고유벡터: 1, $(-1, 1, 0, 0)$; 2, $(0, 0, -1, 1)$; 5, $(-1, 1, 2, 2)$

8.3 연습문제

1. (a)

2. (b)

8.3 컴퓨터 연습문제

4. 고윳값은 $-5, 7, 3$ 이다.

8.4 연습문제

1. A가 대각 지배 행렬이므로 야코비법, 가우스-자이델법이 수렴한다. A가 대칭 정부호 행렬이므로 SOR 법이 수렴한다.

3. (d)

5. (e)

9. (b)

8.4 컴퓨터 연습문제

3. 반복법: 야코비 77, 가우스-자이델 38, SOR 12

7. 둘다 실제 해 $x = -4/11, y = 6/11$ 에 근사하게 수렴한다.