<확률과 랜덤 과정>

연습문제 답안

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1장 연습문제 답안

번호	답안
1.1	(a) 생략 (b) $BC = \{4\}$, $A+B = \{1,3,4,5\}$, $A+C = \{1,2,3,4,5,6\}$ (c) ~ (e) 생략
1.2	생략
1.3	(a) 생략 (b) $\overline{A} \cap B = \{6,8\}$ (c)~(d) 생략
1.4	생략
1.5	(a) 생략 (b) $A+B = \{1,2,5,6,9\}$, $A+\overline{C} = \{1,2,3,4,5,6,7,8,9,10\}$, $A(B+C) = AB = \{1,9\}$ (c)~(d) 생략
1.6	(a) $\int_{-\infty}^{\infty} \cos 6t \cdot \delta(t-3)\,dt = \cos 18$ (b) $\int_{-\infty}^{\infty} e^{-t^2}\delta(t-1)\,dt = e^{-1} = \frac{1}{e}$ (c) $\int_{-\infty}^{\infty} \delta(t-2)\sin \pi t\,dt = \sin 2\pi = 0$ (d) $\int_{-\infty}^{\infty} f(3-t)\,\delta(1-t)\,dt = e^{-1} = f(2)$ (e) $\int_{-\infty}^{\infty} \delta(t+3)\,e^{-t}\,dt = e^3$ (f) $\int_{0}^{\infty} e^x \cos\pi(x+3)\,\delta(x+1)\,dx = 0$
1.7	$x_n = \frac{1}{T_s}$, $f_0 = \frac{1}{T_s}$ 그림 생략
1.8	<첫 번째 그림의 경우> (a) $x(t) = \frac{3}{4} + \sum_{n=1}^{\infty}\left(\left(\frac{8}{n\pi}\right)\sin(n\pi/2) - \left(\frac{4}{n^2\pi^2}\right)(\cos n\pi + \cos(n\pi/2)\right)\cos(n\pi t/2)$ (b) 사인 항(b_n)이 없는 것은 $x(t)$가 우함수이기 때문이다. <두 번째 그림의 경우> (a) $x(t) = \sum_{n=1}^{0}\left(\frac{4}{n\pi}\right)\sin(n\pi t)$ (b) 코사인 항(a_n)이 없는 것은 $x(t)$가 기함수이기 때문이다.
1.9	(a) $P = 2$ [W] (b) $P = x_{-1}^2 + x_1^2 = 2$ [W]
1.10	생략

1.11	(a) $X(f) = \mathrm{sinc}(f)$ (b) $Y(f) = \frac{1}{2}[X(f-f_0) + X(f+f_0)]$ (c) 변조(modulation) 혹은 주파수 이동(frequency shift)
1.12	(a) $X(f) = \frac{1}{2+j2\pi f} + \frac{1}{2-j2\pi f} = \frac{2^2}{2^2+(2\pi f)^2}$ (b) $E_x = 2\int_0^{\infty} e^{-4t}dt = \frac{1}{2}$ (c) $\frac{1}{2}$
1.13	(a) $x(t)*x(t) = \begin{cases} t+2, & -2 \le t < 0 \\ -t+2, & 0 \le t \le 2 \\ 0, & elsewhere \end{cases}$ (b) $Z(f) = 4\,\mathrm{sinc}^2(2f)$
1.14	$X(f) = (-\frac{j}{2})\delta(f-5) + (\frac{j}{2})\delta(f+5)$, 그림 생략
1.15	(a) $H(f) = Ke^{-j2\pi f t_0}$, $\lvert H(f)\rvert = K$ (b) $\theta(f) = -2\pi t_0 f$ (c) 생략
1.16	(a) $y(t) = 400\,\mathrm{sinc}\,20(t-1)$ (b) 생략

2장 연습문제 답안

번호	답안
2.1	(a) $S=\{1,2,3,4,5,6\}$ (b) $A = \{1,2,3\}$, $B = \{2,4,6\}$ (c) $AB = \{2\}$ (d) $A+B = \{1,2,3,4,6\}$ (e) $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{2}$, $P(AB) = \frac{1}{6}$, $P(A+B) = \frac{5}{6}$ (f) 성립한다.
2.2	(a) $P(B\|A) = \frac{3}{51} = \frac{1}{17}$ (b) $P(C\|A) = \frac{4}{51}$ (c) $P(AB) = P(A)P(B\|A) = \frac{1}{221}$
2.3	$P(B) = \frac{1}{3}$
2.4	$P(A) = \frac{6}{36} = \frac{1}{6}$
2.5	(a) $P(B\|A) = \frac{40}{120} = \frac{1}{3}$ (b) $P(A\|B) = \frac{40}{180} = \frac{2}{9}$
2.6	(a) $P(B\|A) = \frac{1}{4}$ (b) $P(C\|D) = \frac{1}{2}$ (c) A와 B는 독립이므로 $P(B\|A) = P(B) = \frac{2}{5}$
2.7	(a) $P(A\|B) = \frac{1}{2}$ (b) $P(AB) = \frac{1}{4} = P(A)P(B) = \frac{1}{2}\times\frac{1}{2}$ 따라서 A와 B는 서로 독립이다.
2.8	(a) $P(A_0\|B_0) = 0.69$ (b) $P(A_1\|B_0) = 0.31$ (c) 송신 심벌은 $A_0 = 0$으로 결정
2.9	(a) $P(A) = \frac{5}{12}$ (b) $P(A)P(B\|A) = \frac{5}{33}$ (c) $P(D) = \frac{77}{132}$ (d) $P(A)P(B\|A) = \frac{5}{36}$

2.10	$(a) P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ (b) $P(AB) = \frac{1}{6}$ (c) $P(AB) = \frac{1}{6} = P(A)P(B) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$, 따라서 A와 B는 서로 독립.
2.11	(a) $P(A) = 0.8$ (b) $P(C_1 \| A) = 0.375$ (c) $P(C_2 \| A) = 0.333$, $P(C_3 \| A) = 0.29$

3장 연습문제 답안

번호	답안
3.1	(a) $S_X = \{0,1,2,3,\cdots,n\}$, X가 갖는 값이 모두 이산 값이므로 이산 랜덤 변수. (b) $P(X=0) = \frac{1}{2^n}$, $P(X=x) = \frac{1}{2^n}$ (c) $P(X \le n) = \sum_{n=0}^{n} \binom{n}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k}$ (d) $F_X(x) = P(X \le x) = \sum_{k=0}^{x} \binom{n}{k} \left(\frac{1}{2}\right)^k \left(\frac{1}{2}\right)^{n-k}$
3.2	(a) $\overline{X} = 0[\mathrm{V}]$ (b) $\sigma_X^2 = E[(X-\overline{X})^2] = A^2$ [W]
3.3	(a)~(b) 생략
3.4	(a) $P(X=1)=0.3$ $P(X=2)=0.21$ $P(X=3)=0.147$ (b) $F_X(3) = P(X \le 3) = 0.657$
3.5	(a) 0.328 (b) P_r\{불합격 저항이 1개\}+P_r\{불합격 저항이 2개\}= 0.401
3.6	(a) $E[X]=0$ (b) $\sigma_X^2 = 3$
3.7	$E[X]=3$, $\sigma_X^2 = 2$
3.8	$p_Y(y) = \frac{1}{8\sqrt{y}} + \frac{1}{2}\delta(y)$, $0 \le y < 4$, $(-2 \le x < 2)$
3.9	$p_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} + \frac{1}{2}\delta(y)$, $y \ge 0$, 그림 생략
3.10	$p_Y(y) = \frac{\frac{a}{\pi}}{a^2+y^2}$, $a > 0$
3.11	(a) $p_X(x) = 0.88\delta(x) + 0.12\delta(x-4)$ (b) $p_Y(y) = 0.35\delta(y) + 0.65\delta(y-1)$ (c) 독립이 아니다. (d) 두 랜덤 변수는 직교한다.
3.12	$p_{XY}(x,y) = p_X(x) \bullet p_Y(y)$ $\therefore$ 독립
3.13	(a) $p_X(x) = \frac{5}{118} y(8-y^3)$, $0 < y < 2$ (b) 독립이 아니다.
3.14	(a) $A=2$ (b) $p_X(x) = 2e^{-2x}$, $p_Y(y) = e^{-y}$ (c) $p_{XY}(x,y) = p_X(x) \cdot p_Y(y)$ $\therefore$ 독립

3.15	(a) $b = \frac{1}{104}$ (b) $p_X(x) = \frac{1}{312}(18x^2 + 54)\,, \quad -2 < x < 2$ $p_Y(y) = \frac{1}{312}(12y^2 + 16)\,, \quad -3 < y < 3$
3.16	$p_Y(y) = \frac{1}{8\sqrt{y}}\,, \ 0 < y < 4$
3.17	(a) $p_Y(y) = \frac{1}{2\sqrt{2\pi}} e^{-(y-1)^2/2 \cdot 4}$ (b) 생략
3.18	(a) $p = \frac{1}{2}$ (b) $E[X] = 2$
3.19	(a) $p_X(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\,, \quad Q(x) = \frac{1}{2\pi}\int_x^\infty e^{-t^2/2}\,dt$ (b) $p_Y(y) = 0.159\,\delta(y+3) + 0.341\,\delta(y+1) + 0.341\,\delta(y-1) + 0.159\,\delta(y-3)$ 그림 생략
3.20	$p_X(x) = \left(\frac{1}{\theta_2 - \theta_1}\right) \cdot \sqrt{1-x^2}\,, \quad x_2 < x < x_1 \leftarrow \cos(\theta_2) < x < \cos(\theta_1)$
3.21	$p_X(x) = \frac{3}{2}e^{-3x}$, $p_Y(y) = 2e^{-4y}$ X와 Y는 독립이 아니다.

4장 연습문제 답안

<table>
<tr><th>번호</th><th>답안</th></tr>
<tr><td>4.1</td><td>
(a) $S_{XY} = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,3),(2,4),(2,5),(2,6)\}$
(b) 생략
(c) $P_X(x) = \frac{21}{36}\delta(x-1)+\frac{15}{36}\delta(x-2)$
$P_Y(y) = \frac{1}{36}\delta(y-1)+\frac{3}{36}\delta(y-2)\frac{5}{36}\delta(y-3)+\frac{7}{36}\delta(y-4)+$
$\frac{9}{36}\delta(y-5)+\frac{11}{36}\delta(y-6)$
(d) $P_{XY}(x,y) = P_X(x)P_Y(y)$
$\frac{1}{36} = P_{XY}(1,1) \neq P_X(x=1)P_Y(y=1) = \frac{21}{36}\times\frac{1}{36}$
X와 Y는 독립이 아니다.
</td></tr>
<tr><td>4.2</td><td>
(a) $S_{XY} = \{(0,0),(0,1),(1,0),(1,1)\}$
(b) 생략
(c) $P_X(x) = \frac{1}{4}\delta(x)+\frac{3}{4}\delta(x-1),\ P_Y(y) = \frac{5}{8}\delta(y)+\frac{3}{8}\delta(y-1)$
(d) 독립이 아니다.
</td></tr>
<tr><td>4.3</td><td>
(a)
<table>
<tr><td>Y / X</td><td>Y=0</td><td>Y=1</td><td>Y=2</td><td>$P_X(x)$</td></tr>
<tr><td>X=0</td><td>$\frac{1}{8}$</td><td>0</td><td>$\frac{1}{4}$</td><td>$\frac{3}{8}$</td></tr>
<tr><td>X=1</td><td>0</td><td>$\frac{1}{8}$</td><td>$\frac{1}{4}$</td><td>$\frac{3}{8}$</td></tr>
<tr><td>X=2</td><td>$\frac{1}{8}$</td><td>0</td><td>$\frac{1}{8}$</td><td>$\frac{2}{8}$</td></tr>
<tr><td>$P_Y(y)$</td><td>$\frac{1}{4}$</td><td>$\frac{1}{8}$</td><td>$\frac{5}{8}$</td><td>1</td></tr>
</table>
(b) 독립이 아니다.
</td></tr>
<tr><td>4.4</td><td>
(a) 생략
(b) $P\{X \le 2.5,\ Y \le 6\} = F_{XY}(2.5, 6) = 0.45$
(c) $P\{X \le 3\} = 0.5$
</td></tr>
<tr><td>4.5</td><td>
(a) $F_{XY}(x,y) = u(x)u(y)(1-e^{-0.5x})(1-e^{-0.5y})$
(b) $P\{X \le 1,\ Y \le 2\} = 0.25$
(c) $P\left\{\frac{1}{2} \le X \le \frac{3}{2}\right\} = 0.306$
</td></tr>
<tr><td>4.6</td><td>
(a) $b=4$
(b) $P\{(0.5\times 4) < X^2+Y^2 \le (0.8\times 4)\} = 0.39$
</td></tr>
<tr><td>4.7</td><td>
(a) $p_X(x) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dy = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$
</td></tr>
</table>

	$p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dx = \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}}$ (b) $\rho \neq 0$ 이면 $p_{XY}(x,y) \neq p_X(x) \cdot p_Y(y)$ $\therefore$ 독립이 아님 $\rho = 0$ 이면 $p_{XY}(x,y) = p_X(x) \cdot p_Y(y)$ $\therefore$ 독립
4.8	$p_X(x) = \int_{-\infty}^{\infty} P_{XY}(x,y)\,dy$ $= 0.15\,\delta(x) + 0.55\,\delta(x-2) + 0.3\,\delta(x-4)$ $p_Y(y) = \int_{-\infty}^{\infty} p_{XY}(x,y)\,dx$ $= 0.22\,\delta(y) + 0.3\,\delta(y-1) + 0.3\,\delta(y-2) + 0.18\,\delta(y-3)$
4.9	$\rho_{XY} = \frac{E\left[a(X-\overline{X})^2\right]}{\sigma_X \lvert a \rvert \sigma_X} = \frac{a}{\lvert a \rvert} = \begin{cases} 1, & a > 0 \\ -1, & a < 0 \end{cases}$
4.10	$p_Y(y) = \begin{cases} \frac{1}{\pi\sqrt{9-y^2}}, & -3 \le y \le 3 \\ 0, & elsewhere \end{cases}$
4.11	$m_Y = E[Y] = 0$ $\sigma_y^2 = E[Y^2]$, $m_Y^2 = \frac{9}{2}$
4.12	(a) $p_Y(y) = \frac{1}{2\sqrt{y}}$, $0 \le y \le 1$ (b) $m_X = E[X] = \int_{-1}^{1} x \cdot \frac{1}{2} dx = [x^2]_{-1}^{1} = 0$ $m_Y = E[Y] = \int_0^1 y \cdot \frac{1}{2\sqrt{y}} dy = \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$ $\sigma_X^2 = E\left[(X-m_X)^2\right] = E\left[X^2\right] = \frac{1}{3}$ $\sigma_Y^2 = 0.089$ (c) $\rho_{XY} = 0$ (d) X와 Y는 $uncorrelated.$
4.13	$p_Y(y) = \frac{y^{\frac{3}{2}}}{3\sqrt{2\pi}\,\sigma_X} \cdot e^{\frac{-y^{\frac{2}{3}}}{2\sigma_X^2}}$
4.14	$p_{X_1X_2}(x_1,x_2) = (1-a_1)(1-a_2)\,a_1^{x_1} a_2^{x_2}$ $\left(\leftarrow \sum_{x_3=0}^{\infty} a_3^{x_3} = \frac{1}{1-a_3}\right)$ $p_{X_1}(x_1) = \sum_{x_2=0}^{\infty} p_{X_1X_2}(x_1, x_2) = (1-a_1)\,a_1^{x_1}$

5장 연습문제 답안

번호	답안
5.1	(a) $E[\boldsymbol{X}] = 0$ (b) $R_{XX}(t_1, t_2) = R_{XX}(\tau)$, $(\leftarrow \tau = t_2 - t_1)$ (c) $\boldsymbol{X}(t)$는 WSS
5.2	(a) $p_X(x) = \dfrac{1}{2\lvert\cos\omega_0 t\rvert}$, $-\lvert\cos\omega_0 t\rvert \le X \le \lvert\cos\omega_0 t\rvert$ $E[X] = \displaystyle\int_{-\lvert\cos\omega_0 t\rvert}^{\lvert\cos\omega_0 t\rvert} x\, p_X(x)\, dx = \int_{-\lvert\cos\omega_0 t\rvert}^{\lvert\cos\omega_0 t\rvert} x \frac{1}{2\lvert\cos\omega_0 t\rvert}\, dx = 0$ (b) $R_{XX}(t_1, t_2) = E[X(t_1)X(t_2)]$ $= \dfrac{1}{3}(\cos\omega_0 t_1)(\cos\omega_0 t_2)$
5.3	생략
5.4	(a) 생략 (b) $p_X(x) = \dfrac{1}{2}\delta(x-1) + \dfrac{1}{2}\delta(x+1)$
5.5	(a) $p_X(x) = \dfrac{1}{T}\displaystyle\int_0^T \left[\left(\frac{\tau}{T}\right)\delta(x-A) + \left(\frac{T-\tau}{T}\right)\delta(x+A)\right] d\alpha$ (b) $E[X] = \left(\dfrac{A}{T}\right)(2\tau - T)$ (c) $\sigma_X^2 = \dfrac{4\tau A^2(T-\tau)}{T^2}$ (d) $R_{XX}(t, t+\tau) = \displaystyle\int_0^T x(t+\alpha)\, x(t+\tau+\alpha)\left(\frac{1}{T}\right) d\alpha$ (e) $\boldsymbol{X}(t)$는 WSS
5.6	(a) $E[\boldsymbol{Y}(t)] = \left(\dfrac{A^2}{2}\right) + 3$ (b) $R_{YY}(t, t+\tau) = R_{YY}(\tau)$ (c) $\boldsymbol{Y}(t)$는 WSS
5.7	(a) $E[X(t)] = E_\omega[0] = 0$ (b) $R_{XX}(t, t+\tau) = R_{XX}(\tau)$ (c) $\boldsymbol{X}(t)$는 WSS
5.8	자기 상관 함수가 되기 위해서는 다음 세 가지 조건을 만족하면 된다. ① $R_{XX}(\tau) \ge 0$ ② $R_{XX}(\tau) = R_{XX}(-\tau)$ ③ $\lvert R_{XX}(\tau)\rvert \le R_{XX}(0)$ (a) ① ② ③ 모두 만족 (b) $R_{XX}(-2\pi) = e^{2\pi}\cos 2\pi = e^{2\pi} > R_{XX}(0) = 1$ ③을 만족시키지 못하므로 자기 상관 함수가 아니다. (c) ① ② ③ 모두 만족

5.9	$\int_0^T g_{t_0}(t-\epsilon)\, g_{t_0}(t+\tau-\epsilon)\, d\epsilon = \begin{cases} t_0 - \tau,\ 0 \le \tau < t_0 \\ t_0 + \tau,\ -t_0 \le \tau < 0 \end{cases}$ $= t_0\left(1-\frac{\lvert\tau\rvert}{t_0}\right),\ \lvert\tau\rvert < t_0$ $= t_0\left(1-\frac{\lvert\tau\rvert}{t_0}\right)[u(\tau+t_0)-u(\tau-t_0)]$ 단 $\sigma_a^2 = E[a^2]-(E[a])^2$
5.10	(a) $E[\boldsymbol{X}(t)] = 0$ (b) $R_{XX}(t,t+\tau) = \sigma_X^2 \cos 2\pi f_c t \cos 2\pi f_c(t+\tau) + \sigma_Y^2 \sin 2\pi f_c t \sin 2\pi f_c(t+\tau)$ ($\leftarrow$ X와 Y는 독립이므로 $E[XY] = 0$) 따라서, $\boldsymbol{X}(t)$는 WSS가 아니다. (c) $R_{XX}(t,t+\tau) = R_{XX}(\tau)$, $\boldsymbol{X}(t)$는 WSS
5.11	$R_{YY}(t,t+\tau) = R_{X'X'}(\tau) + R_{XX}(\tau)$
5.12	$E[X(t)] = \frac{1}{2}x_1(t) + \frac{1}{2}x_2(t) = \frac{3}{2} + \frac{1}{2}\cos 3t$ $R_{XX}(t,t+\tau) = \frac{9}{2} + \frac{1}{2}\cos(3t)\cos 3(t+\tau)$ 따라서 WSS가 아니다.
5.13	생략
5.14	$\boldsymbol{Z}(t)$는 비 정적 랜덤 과정
5.15	(a) 생략 (b) $\boldsymbol{X}(t)$와 $\boldsymbol{N}(t)$는 WSS (c) $\boldsymbol{Y}(t)$와 $\boldsymbol{N}(t)$는 결합 WSS
5.16	$p_{\boldsymbol{X}(t)\boldsymbol{X}(t+1)}(x_0, x_1) = \frac{1}{\sqrt{3\pi^2}}\, e^{-\frac{2}{3}(x_0^2 - x_0 x_1 + x_1^2)}$
5.17	평균이 t의 함수이므로 WSS 복소 랜덤 과정이 아니다.

6장 연습문제 답안

번호	답안
6.1	(a) 생략 (b) $m_X(t) = 0$ (c) $R_{XX}(t, t+\tau) = \frac{1}{3}$ (d) 평균과 자기 상관함수 모두 상수이므로 WSS (e) $S_X(f) = \frac{1}{3}\delta(f)$
6.2	(a) 생략 (b) $X(t)$와 $Y(t)$는 결합 WSS 랜덤 과정 (c) 생략
6.3	(a) 생략 (b) $S_X(f) = S_Y(f) = N_0, \quad \lvert f \rvert < W$ (c) 생략
6.4	(a) $R_{UU}(t, t+\tau) = 4R_{VV}(\tau)$ (b) $R_{UV}(t, t+\tau) = 2R_{XX}(\tau) + 3R_{YY}(\tau)$
6.5	(a) 생략 (b) 비정적 랜덤과정 (c) $m_X(t) = \left(\frac{1}{100t}\right)[\sin 100t - 0]$ $R_{XX}(t, t+\tau) = \left(\frac{1}{200\tau}\right)\sin 100\tau + \frac{1}{200(2t+\tau)} \cdot \sin 100(2t+\tau)$ (d) 타당
6.6	(a) $R_{XX}(\tau) = (N_0 W)\mathrm{sinc}\,(2W\tau)$ $\therefore \tau = \pm \frac{k}{2W}$ 일 때 $X(t)$와 $X(t+\tau)$ 는 uncorrelated $(k = 1, 2, \cdots\cdots)$ (b) $W \to \infty$ $\lim_{W \to \infty} R_{XX}(\tau) = \frac{N_0}{2}\delta(\tau)$ $\to X(t)$는 백색(white) 특성을 가짐
6.7	생략
6.8	(a) $m_Z(t) = 0$ (b) $R_{ZZ}(t) = \sigma^2 \cos 2\pi f_0 \tau = R_{ZZ}(\tau)$ (c) (a), (b)로부터 $Z(t)$는 WSS 랜덤 과정 (d) $S_Z(f) = \frac{\sigma^2}{2}[\delta(f - f_0) + \delta(f + f_0)]$

6.9	(a) $m_X(t) = 0$ (b) $R_{XX}(t, t+\tau) = \sum_{n=-\infty}^{\infty} g(t-nT)g(t+\tau-nT$ (c) 생략 (d) $S_X(f) = \frac{1}{T}\{G(f)\cdot G^*(f)\} = \frac{\lvert G(f)\rvert^2}{T}$ $(\leftarrow G(f) = \mathcal{F}\{g(\tau)\})$
6.10	(a) $m_X(t) = \sum_{n=-\infty}^{\infty}\frac{1}{2}g(t-nT) = \frac{1}{2}\sum_{n=-\infty}^{\infty}g(t-nT)$ (b) $R_{XX}(t, t+\tau) = \sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}R_{AA}(n-m)g(t-nT)g(t+\tau-mT)$ (c) 생략 (d) $S_X(f) = \frac{\lvert G(f)\rvert^2}{T}\sum_{k=-\infty}^{\infty}R_{AA}(k)\cdot e^{-j2\pi fkT}$
6.11	(a) $m_X(t)$는 주기가 $T=\frac{1}{f_c}$ 인 주기함수 (b) $R_{XX}(t, t+\tau)$는 주기가 $\left(\frac{1}{2f_c}\right)$인 주기함수 (c) (a), (b)의 결과로부터 $X(t)$는 주기적 정적 랜덤과정 (d) $S_X(f) = \frac{1}{4}\{\delta(f-f_c)+\delta(f+f_c)\}$
6.12	(a) $m_X(t) = \frac{1}{2}$ (b) $R_{XX}(t, t+\tau) = \frac{2}{3}$ (c) $S_X(f) = \frac{2}{3}\delta(f)$
6.13	(a) $P[X(t)=1] = \frac{1}{2}$, $P[X(t)=-1] = \frac{1}{2}$ (b) $m_X(t) = 0$ (c) $R_{XX}(t, t+\tau) = e^{-2\alpha\lvert\tau\rvert}$ (d) $S_X(f) = \frac{4\alpha}{4\alpha^2+4\pi^2f^2}$
6.14	(a) $E[Z(t)] = M_XM_Y$ (b) $R_{XX}(t, t+\tau) = R_{XX}(\tau)\cdot R_{YY}(\tau)$ (c) $S_Z(f) = S_X(f) * S_Y(f)$
6.15	$S_Y(f) = \left(\frac{13}{4}A^2+27\right)\delta(f) + \frac{A^2}{16}[\delta(f-2f_0)+\delta(f+2f_0)]$
6.16	$S_X(f) = \left(\frac{\sigma^2}{2}\right)\{\delta(f-f_c)+\delta(f+f_c)\}$
6.17	$S_Y(f) = S_X(f) + S_N(f)$

7장 연습문제 답안

번호	답안
7.1	$Y(t) = \left(\frac{1}{1+(2\pi f_0)^2}\right)[\sin(2\pi f_o t+\theta)-(2\pi f_0)\cos(2\pi f_0 t+\theta)]$
7.2	(a) $S_Y(f) = \left(\frac{N_0}{2}\right)\frac{1}{1+(2\pi fRC)^2}$, 그래프 생략 (b) $R_{YY}(\tau) = \left(\frac{N_0}{4RC}\right)e^{-\lvert\tau\rvert/RC}$, 그래프 생략
7.3	(a) 생략 (b) $\lvert H(f)\rvert^2 = 8\pi^2 f^2(1+\cos 2\pi f t_d)$, 그래프 생략 (c) $S_Y(f) = S_X(f)\, 8\pi^2 f^2(1+\cos 2\pi f t_d)$ (d) $f=0, \frac{1}{2t_d}, \frac{3\pi}{2t_d}, \cdots$
7.4	(a) $Z(t)$는 주기적 정적 랜덤 과정 (b) 생략
7.5	(a) $R_{YY}(\tau) = R_{XX}(\tau) * \delta(\tau) = R_{XX}(\tau)$ (b) $S_Y(f) = S_X(f)$
7.6	$S_Y(f) = S_X(f)+S_{X'}(f)$
7.7	(a) $m_X(t)=E[Xe^{j2\pi f_0 t}]=m_X e^{j2\pi f_0 t}$ $m_Y(t)=E[Ye^{j2\pi f_0 t}]=m_Y e^{j2\pi f_0 t}$ (b) $R_{XY}(t,t+\tau) = R_{XY}(\tau)$ (c) $X(t)$와 $Y(t)$는 모두 비정적 랜덤 과정
7.8	(a) $S_Y(f) = \left(\frac{4\alpha\beta^2}{\beta^2-4\alpha^2}\right)\left\{\frac{1}{4\alpha^2+4\pi^2 f^2} - \frac{1}{\beta^2+4\pi^2 f^2}\right\}$ (b) $R_{YY}(\tau) = \left(\frac{1}{\beta^2-4\alpha^2}\right)\left[\beta^2 e^{-2\alpha\lvert\tau\rvert} - 2\alpha\beta e^{-\beta\lvert\tau\rvert}\right]$
7.9	(a) $R_{XX}(\tau) = \frac{N_0\sin(2\pi W\tau)}{2\pi\tau}$, 그래프 생략 (b) $P_{X(t)} = R_{XX}(0) = N_0 W$ (c) $R_{YY}(\tau) = (2N_0\pi^2)\left(\left(\frac{W^2}{\pi\tau}\right)-\frac{1}{2\pi^3\tau^3}\right)\sin(2\pi\tau W)+\left(\frac{W}{\pi^2\tau^2}\right)$ (d) $S_Y(f) = 2N_0\pi^2 f^2$, $\lvert f\rvert \le W$ (e) 식 ①에서 $R_{YY}(0) = (2N_0\pi^2)\int_{-W}^{W} f^2\,df = \frac{4N_0\pi^2 W^3}{3}$

7.10	(a) $R_{YX}(\tau) = \dfrac{N_0}{2} e^{-\tau},\ \tau > 0$ $\left(\leftarrow \text{(표 } 1-3)\ e^{-\alpha t} u(t)\ ,\ \alpha > 0 \leftrightarrow \dfrac{1}{\alpha + j2\pi f}\right)$ (b) $S_{YX}(f) = \left(\dfrac{N_0}{2}\right) \dfrac{1}{1 + j2\pi f}$ (c) $R_Y(\tau) = \dfrac{N_0}{4} e^{-\lvert\tau\rvert} \quad \left(e^{-\alpha\lvert t\rvert},\ \alpha > 0 \quad \leftrightarrow \quad \dfrac{2\alpha}{\alpha^2 + (2\pi f)^2}\right)$ (d) $P_{\boldsymbol{Y}(t)} = R_{YY}(0) = \dfrac{N_0}{4}$
7.11	(a) $S_Z(f) = \lvert 1 - H(f)\rvert^2 S_X(f)$ (b) $E[Z^2(t)] = R_{XX}(0) + R_{YY}(0) - 2R_{XY}(0)$