5장 연습문제 정답

01.

(a) 행렬 A의 크기 : 3×3

행렬 B의 크기 : 4×4

(b) $a_{13}=-1$, $a_{23}=-5$, $a_{31}=-1$, $b_{14}=3$, $b_{22}=7$, $b_{33}=7$, $b_{43}=3$

(c) 행렬 A의 3행 : $\begin{bmatrix} -1 & 5 & 1 \end{bmatrix}$

행렬 B의 3행 : $\begin{bmatrix} 0 & 9 & 7 & 6 \end{bmatrix}$

(d) 행렬 A의 1열 : $\begin{bmatrix} 1 \\ -5 \\ -1 \end{bmatrix}$

행렬 B의 1열 : $\begin{bmatrix} 6 \\ 4 \\ 0 \\ 5 \end{bmatrix}$

(e) a_{22}, b_{31}

(f) a_{12}, a_{32}, b_{41}, b_{42}

02.

(a) $A+A=\begin{bmatrix} -5 & 0 \\ 1 & 2 \end{bmatrix}+\begin{bmatrix} -5 & 0 \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} -10 & 0 \\ 2 & 4 \end{bmatrix}$

(b) 행렬 A의 크기는 2×2이고 행렬 B의 크기는 3×3이므로 연산할 수 없다.

(c) 행렬 A의 크기는 2×2이고 행렬 D의 크기는 3×2이므로 연산할 수 없다.

(d) $B+B=\begin{bmatrix} 0 & 4 & 2 \\ 7 & 1 & 5 \\ -2 & -7 & 9 \end{bmatrix}+\begin{bmatrix} 0 & 4 & 2 \\ 7 & 1 & 5 \\ -2 & -7 & 9 \end{bmatrix}=\begin{bmatrix} 0 & 8 & 4 \\ 14 & 2 & 10 \\ -4 & -14 & 18 \end{bmatrix}$

(e) 행렬 B의 크기는 3×3이고 행렬 C의 크기는 3×2이므로 연산할 수 없다.

(f) 행렬 A의 크기는 2×2이고 행렬 C의 크기는 3×2이므로 연산할 수 없다.

(g) 행렬 B의 크기는 3×3이고 행렬 A의 크기는 2×2이므로 연산할 수 없다.

(h) $B-B=\begin{bmatrix} 0 & 4 & 2 \\ 7 & 1 & 5 \\ -2 & -7 & 9 \end{bmatrix}-\begin{bmatrix} 0 & 4 & 2 \\ 7 & 1 & 5 \\ -2 & -7 & 9 \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(i) 행렬 B의 크기는 3×3이고 행렬 D의 크기는 3×2이므로 연산할 수 없다.

(j) 행렬 C의 크기는 3×2이고 행렬 B의 크기는 3×3이므로 연산할 수 없다.

03.

(a) 연산할 수 없다.

(b) $C+D=\begin{bmatrix} 5 & 5 \\ 10 & -6 \\ -2 & 12 \end{bmatrix}$

(c) 연산할 수 없다.

(d) $D+C=\begin{bmatrix} 5 & 5 \\ 10 & -6 \\ -2 & 12 \end{bmatrix}$

(e) $D+D=\begin{bmatrix} -2 & 4 \\ 10 & -4 \\ 0 & 6 \end{bmatrix}$

(f) $C-C=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$

(g) $C-D=\begin{bmatrix}7&1\\0&-2\\-2&6\end{bmatrix}$

(h) 연산할 수 없다.

(i) 연산할 수 없다.

(j) $D-D=\begin{bmatrix}0&0\\0&0\\0&0\end{bmatrix}$

04.

(a) 연산식의 앞의 행렬 A의 열의 크기(2)와 뒤의 행렬 A의 행의 크기(3)가 다르므로 연산할 수 없다.

(b) $AB=\begin{bmatrix}1&3\\5&1\\8&0\end{bmatrix}\times\begin{bmatrix}7&1\\4&5\end{bmatrix}=\begin{bmatrix}(1\times7)+(3\times4)&(1\times1)+(3\times5)\\(5\times7)+(1\times4)&(5\times1)+(1\times5)\\(8\times7)+(0\times4)&(8\times1)+(0\times5)\end{bmatrix}=\begin{bmatrix}19&16\\39&10\\56&8\end{bmatrix}$

(c) 연산식의 앞의 행렬 A의 열의 크기(2)와 뒤의 행렬 C의 행의 크기(3)가 다르므로 연산할 수 없다.

(d) $AD=\begin{bmatrix}1&3\\5&1\\8&0\end{bmatrix}\times\begin{bmatrix}0&1&2\\0&3&4\end{bmatrix}=\begin{bmatrix}(1\times0)+(3\times0)&(1\times1)+(3\times3)&(1\times2)+(3\times4)\\(5\times0)+(1\times0)&(5\times1)+(1\times3)&(5\times2)+(1\times4)\\(8\times0)+(0\times0)&(8\times1)+(0\times3)&(8\times2)+(0\times4)\end{bmatrix}=\begin{bmatrix}0&10&14\\0&8&14\\0&8&16\end{bmatrix}$

(e) 연산식의 앞의 행렬 B의 열의 크기(2)와 뒤의 행렬 A의 행의 크기(3)가 다르므로 연산할 수 없다.

(f) $2BD=2\begin{bmatrix}7&1\\4&5\end{bmatrix}\times\begin{bmatrix}0&1&2\\0&3&4\end{bmatrix}=\begin{bmatrix}14&2\\8&10\end{bmatrix}\times\begin{bmatrix}0&1&2\\0&3&4\end{bmatrix}$

$=\begin{bmatrix}(14\times0)+(2\times0)&(14\times1)+(2\times3)&(14\times2)+(2\times4)\\(8\times0)+(10\times0)&(8\times1)+(10\times3)&(8\times2)+(10\times4)\end{bmatrix}=\begin{bmatrix}0&20&36\\0&38&56\end{bmatrix}$

(g) $C-AD=\begin{bmatrix}4&5&3\\8&0&1\\2&7&2\end{bmatrix}-\begin{bmatrix}1&3\\5&1\\8&0\end{bmatrix}\times\begin{bmatrix}0&1&2\\0&3&4\end{bmatrix}$

$=\begin{bmatrix}4&5&3\\8&0&1\\2&7&2\end{bmatrix}-\begin{bmatrix}(1\times0)+(3\times0)&(1\times1)+(3\times3)&(1\times2)+(3\times4)\\(5\times0)+(1\times0)&(5\times1)+(1\times3)&(5\times2)+(1\times4)\\(8\times0)+(0\times0)&(8\times1)+(0\times3)&(8\times2)+(0\times4)\end{bmatrix}$

$=\begin{bmatrix}4&5&3\\8&0&1\\2&7&2\end{bmatrix}-\begin{bmatrix}0&10&14\\0&8&14\\0&8&16\end{bmatrix}=\begin{bmatrix}4&-5&-11\\8&-8&-13\\2&-1&-14\end{bmatrix}$

(h) $A+A$의 열의 크기(2)와 행렬 C의 행의 크기(3)가 다르므로 연산할 수 없다.

(i) $DA+B=\begin{bmatrix}0&1&2\\0&3&4\end{bmatrix}\times\begin{bmatrix}1&3\\5&1\\8&0\end{bmatrix}+\begin{bmatrix}7&1\\4&5\end{bmatrix}$

$=\begin{bmatrix}(0\times1)+(1\times5)+(2\times8)&(0\times3)+(1\times1)+(2\times0)\\(0\times1)+(3\times5)+(4\times8)&(0\times3)+(3\times1)+(4\times0)\end{bmatrix}+\begin{bmatrix}7&1\\4&5\end{bmatrix}$

$=\begin{bmatrix}21&1\\47&3\end{bmatrix}+\begin{bmatrix}7&1\\4&5\end{bmatrix}=\begin{bmatrix}28&2\\51&8\end{bmatrix}$

05.

풀이 생략

06.

(a) $3A = \begin{bmatrix} 3 & 9 \\ 15 & 3 \\ 24 & 0 \end{bmatrix}$

(b) $6B = \begin{bmatrix} 42 & 6 \\ 24 & 30 \end{bmatrix}$

(c) $4C = \begin{bmatrix} 16 & 20 & 12 \\ 32 & 0 & 4 \\ 8 & 28 & 8 \end{bmatrix}$

(d) $-5D = \begin{bmatrix} 0 & -5 & -10 \\ 0 & -15 & -20 \end{bmatrix}$

07.

(a) 연산할 수 없다.

(b) $CC = \begin{bmatrix} 62 & 41 & 23 \\ 34 & 47 & 26 \\ 68 & 24 & 17 \end{bmatrix}$

(c) 연산할 수 없다.

(d) $DC = \begin{bmatrix} 12 & 14 & 5 \\ 32 & 28 & 11 \end{bmatrix}$

(e) $DA = \begin{bmatrix} 21 & 1 \\ 47 & 3 \end{bmatrix}$

(f) 연산할 수 없다.

(g) $BD - DC = \begin{bmatrix} -12 & -4 & 13 \\ -32 & -9 & 17 \end{bmatrix}$

(h) $5CA + AB = \begin{bmatrix} 284 & 101 \\ 119 & 130 \\ 321 & 73 \end{bmatrix}$

(i) 연산할 수 없다.

08.

$A^T = \begin{bmatrix} 1 & 5 & 8 \\ 5 & 3 & 0 \\ 8 & 0 & 7 \end{bmatrix}$ $\quad \therefore$ $A = A^T$ 이므로 A는 대칭행렬

$B^T = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 1 \\ 8 & 7 & 5 \end{bmatrix}$ $\quad \therefore$ $B \neq B^T$ 이므로 B는 대칭행렬이 아니다.

$C^T = \begin{bmatrix} 0 & 0 & 8 & -5 \\ 1 & -3 & 2 & 1 \\ 1 & 1 & 3 & 1 \\ 0 & -8 & 7 & 5 \end{bmatrix}$ $\quad \therefore$ $C \neq C^T$ 이므로 C는 대칭행렬이 아니다.

$D^T = \begin{bmatrix} 1 & 9 & 4 & 0 \\ 9 & 2 & -2 & -8 \\ 4 & -2 & -3 & 1 \\ 0 & -8 & 1 & 9 \end{bmatrix}$ $\quad \therefore$ $D \neq D^T$ 이므로 D는 대칭행렬이 아니다.

09.

$$A^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & -1 & 7 \\ 0 & 2 & 6 & -3 \\ -1 & 6 & 3 & 5 \\ 7 & -3 & 5 & 4 \end{bmatrix}$$, A는 대칭행렬

$$B^{\mathrm{T}} = \begin{bmatrix} 5 & 3 & 1 & -2 \\ 0 & 4 & -7 & -1 \\ -1 & 8 & 3 & 2 \\ -5 & 6 & 9 & 0 \end{bmatrix}$$, B는 대칭행렬이 아니다.

$$C^{\mathrm{T}} = \begin{bmatrix} 3 & 1 & 5 & 7 \\ 1 & -2 & -4 & 2 \\ 2 & 6 & 1 & 5 \\ -7 & -3 & 8 & 4 \end{bmatrix}$$, C는 대칭행렬이 아니다.

$$D^{T} = \begin{bmatrix} -3 & 8 & -4 & 1 \\ 8 & 7 & 2 & 2 \\ -4 & 2 & 6 & 5 \\ 1 & 2 & 5 & 8 \end{bmatrix}$$, D는 대칭행렬

10.

(a) $A \vee C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$A \wedge C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(b) $A \odot A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (1 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 1) \vee (0 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

$A \odot B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \\ (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 1) & (0 \wedge 0) \vee (0 \wedge 1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$A \odot C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (0 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$B \odot D = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} (0 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 0) \vee (0 \wedge 1) \\ (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \vee (1 \wedge 1) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$

$C \odot A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (0 \wedge 1) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 1) \vee (0 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$C \odot B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} (0 \wedge 0) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \\ (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 1) & (0 \wedge 0) \vee (0 \wedge 1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$C \odot C = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (0 \wedge 0) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (0 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 0) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$D \odot A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (1 \wedge 1) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 1) \vee (0 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 0) \\ (0 \wedge 1) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 0) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

$D \odot B = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 1) & (1 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 0) \vee (1 \wedge 1) \\ (0 \wedge 0) \vee (0 \wedge 1) & (0 \wedge 1) \vee (0 \wedge 1) & (0 \wedge 0) \vee (0 \wedge 1) \\ (0 \wedge 0) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 0) \vee (1 \wedge 1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$D \odot C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} (1 \wedge 0) \vee (1 \wedge 0) & (1 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (0 \wedge 0) & (0 \wedge 1) \vee (0 \wedge 0) \\ (0 \wedge 0) \vee (1 \wedge 0) & (0 \wedge 1) \vee (1 \wedge 0) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

11.

(a) $(C\odot B)\vee A=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

(b) $(C\odot B)\wedge A=\left(\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\odot\begin{bmatrix}0 & 0\\ 1 & 1\\ 1 & 1\end{bmatrix}\right)\wedge\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\}$

$$=\begin{bmatrix}(1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1) & (1\wedge 0)\vee(1\wedge 1)\vee(0\wedge 1)\\ (0\wedge 0)\vee(0\wedge 1)\vee(1\wedge 1) & (0\wedge 0)\vee(0\wedge 1)\vee(1\wedge 1)\end{bmatrix}\wedge\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\wedge\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}$$

(c) $(B\odot C)\vee D=\left(\begin{bmatrix}0 & 0\\ 1 & 1\\ 1 & 1\end{bmatrix}\odot\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)\vee\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$

$$=\begin{bmatrix}(0\wedge 1)\vee(0\wedge 0) & (0\wedge 1)\vee(0\wedge 0) & (0\wedge 0)\vee(0\wedge 1)\\ (1\wedge 1)\vee(1\wedge 0) & (1\wedge 1)\vee(1\wedge 0) & (1\wedge 0)\vee(1\wedge 1)\\ (1\wedge 1)\vee(1\wedge 0) & (1\wedge 1)\vee(1\wedge 0) & (1\wedge 0)\vee(1\wedge 1)\end{bmatrix}\vee\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}\vee\begin{bmatrix}1 & 1 & 0\\ 0 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 0\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

(d) $(B\odot C)\wedge D=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}$

12.

(a) $\det(A)=\{8\times(-2)\}-\{3\times(-2)\}=(-16)-(-6)=-10$

(b) $\det(B)=0$

(c) $\det(C)=20$

13.

(a)

행렬 A의 소행렬 및 소행렬식

$M_{11}=\begin{bmatrix}5 & 3\\ 4 & 1\end{bmatrix}$ $\quad\det(M_{11})=(5\times 1)-(3\times 4)=-7$

$M_{12}=\begin{bmatrix}-3 & 3\\ 0 & 1\end{bmatrix}$ $\quad\det(M_{12})=\{(-3)\times 1\}-(3\times 0)=-3$

$M_{13}=\begin{bmatrix}-3 & 5\\ 0 & 4\end{bmatrix}$ $\quad\det(M_{13})=\{(-3)\times 4\}-(5\times 0)=-12$

$M_{21}=\begin{bmatrix}1 & -1\\ 4 & 1\end{bmatrix}$ $\quad\det(M_{21})=(1\times 1)-\{(-1)\times 4\}=5$

$M_{22}=\begin{bmatrix}2 & -1\\ 0 & 1\end{bmatrix}$ $\quad\det(M_{22})=(2\times 1)-\{(-1)\times 0\}=2$

$M_{23}=\begin{bmatrix}2 & 1\\ 0 & 4\end{bmatrix}$ $\quad\det(M_{23})=(2\times 4)-(1\times 0)=8$

$M_{31}=\begin{bmatrix}1 & -1\\ 5 & 3\end{bmatrix}$ $\quad\det(M_{31})=(1\times 3)-\{(-1)\times 5\}=8$

$M_{32}=\begin{bmatrix}2 & -1\\ -3 & 3\end{bmatrix}$ $\quad\det(M_{32})=(2\times 3)-\{(-1)\times(-3)\}=3$

$M_{33}=\begin{bmatrix}2 & 1\\ -3 & 5\end{bmatrix}$ $\quad\det(M_{33})=(2\times 5)-\{1\times(-3)\}=13$

행렬 B의 소행렬 및 소행렬식

$M_{11} = \begin{bmatrix} 5 & -4 \\ 6 & 9 \end{bmatrix}$ $\det(M_{11}) = (5\times 9) - \{(-4)\times 6\} = 69$

$M_{12} = \begin{bmatrix} 0 & -4 \\ 3 & 9 \end{bmatrix}$ $\det(M_{12}) = (0\times 9) - \{(-4)\times 3\} = 12$

$M_{13} = \begin{bmatrix} 0 & 5 \\ 3 & 6 \end{bmatrix}$ $\det(M_{13}) = (0\times 6) - (5\times 3) = -15$

$M_{21} = \begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$ $\det(M_{21}) = (2\times 9) - (3\times 6) = 0$

$M_{22} = \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}$ $\det(M_{22}) = (1\times 9) - (3\times 3) = 0$

$M_{23} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ $\det(M_{31}) = (1\times 6) - (2\times 3) = 0$

$M_{31} = \begin{bmatrix} 2 & 3 \\ 5 & -4 \end{bmatrix}$ $\det(M_{31}) = \{2\times(-4)\} - (3\times 5) = -23$

$M_{32} = \begin{bmatrix} 1 & 3 \\ 0 & -4 \end{bmatrix}$ $\det(M_{32}) = \{1\times(-4)\} - (3\times 0) = -4$

$M_{33} = \begin{bmatrix} 1 & 2 \\ 0 & 5 \end{bmatrix}$ $\det(M_{33}) = (1\times 5) - (2\times 0) = 5$

행렬 C의 소행렬 및 소행렬식

$M_{11} = \begin{bmatrix} 2 & 4 \\ 1 & 7 \end{bmatrix}$ $\det(M_{11}) = (2\times 7) - (4\times 1) = 10$

$M_{12} = \begin{bmatrix} 0 & 4 \\ 0 & 7 \end{bmatrix}$ $\det(M_{12}) = (0\times 7) - (4\times 0) = 0$

$M_{13} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$ $\det(M_{13}) = (0\times 1) - (2\times 0) = 0$

$M_{21} = \begin{bmatrix} -6 & 3 \\ 1 & 7 \end{bmatrix}$ $\det(M_{21}) = \{(-6)\times 7\} - (3\times 1) = -45$

$M_{22} = \begin{bmatrix} 0 & 3 \\ 0 & 7 \end{bmatrix}$ $\det(M_{22}) = (0\times 7) - (3\times 0) = 0$

$M_{23} = \begin{bmatrix} 0 & -6 \\ 0 & 1 \end{bmatrix}$ $\det(M_{23}) = (0\times 1) - \{(-6)\times 0\} = 0$

$M_{31} = \begin{bmatrix} -6 & 3 \\ 2 & 4 \end{bmatrix}$ $\det(M_{31}) = \{(-6)\times 4\} - (3\times 2) = -30$

$M_{32} = \begin{bmatrix} 0 & 3 \\ 0 & 4 \end{bmatrix}$ $\det(M_{32}) = (0\times 4) - (3\times 0) = 0$

$M_{33} = \begin{bmatrix} 0 & -6 \\ 0 & 2 \end{bmatrix}$ $\det(M_{33}) = (0\times 2) - \{(-6)\times 0\} = 0$

행렬 D의 소행렬 및 소행렬식

$M_{11} = \begin{bmatrix} -3 & 6 \\ 1 & -1 \end{bmatrix}$ $\det(M_{11}) = \{(-3)\times(-1)\} - (6\times 1) = -3$

$M_{12} = \begin{bmatrix} 5 & 6 \\ -2 & -1 \end{bmatrix}$ $\det(M_{12}) = \{5\times(-1)\} - \{6\times(-2)\} = 7$

$M_{13} = \begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}$ $\det(M_{13}) = (5\times 1) - \{(-3)\times(-2)\} = -1$

$M_{21} = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$ $\det(M_{21}) = \{2\times(-1)\} - (4\times 1) = -6$

$M_{22} = \begin{bmatrix} 1 & 4 \\ -2 & -1 \end{bmatrix}$ $\det(M_{22}) = \{1\times(-1)\} - \{4\times(-2)\} = 7$

$$M_{23}=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix} \qquad \det(M_{23})=(1\times 1)-\{2\times(-2)\}=5$$

$$M_{31}=\begin{bmatrix}2 & 4\\ -3 & 6\end{bmatrix} \qquad \det(M_{31})=(2\times 6)-\{4\times(-3)\}=24$$

$$M_{32}=\begin{bmatrix}1 & 4\\ 5 & 6\end{bmatrix} \qquad \det(M_{32})=(1\times 6)-(4\times 5)=-14$$

$$M_{33}=\begin{bmatrix}1 & 2\\ 5 & -3\end{bmatrix} \qquad \det(M_{33})=\{1\times(-3)\}-(2\times 5)=-13$$

행렬 E의 소행렬 및 소행렬식

$$M_{11}=\begin{bmatrix}8 & 4\\ 0 & 1\end{bmatrix} \qquad \det(M_{11})=(8\times 1)-(4\times 0)=8$$

$$M_{12}=\begin{bmatrix}6 & 4\\ -1 & 1\end{bmatrix} \qquad \det(M_{12})=(6\times 1)-\{4\times(-1)\}=10$$

$$M_{13}=\begin{bmatrix}6 & 8\\ -1 & 0\end{bmatrix} \qquad \det(M_{13})=(6\times 0)-\{8\times(-1)\}=8$$

$$M_{21}=\begin{bmatrix}4 & 2\\ 0 & 1\end{bmatrix} \qquad \det(M_{21})=(4\times 1)-(2\times 0)=4$$

$$M_{22}=\begin{bmatrix}3 & 2\\ -1 & 1\end{bmatrix} \qquad \det(M_{22})=(3\times 1)-\{2\times(-1)\}=5$$

$$M_{23}=\begin{bmatrix}3 & 4\\ -1 & 0\end{bmatrix} \qquad \det(M_{23})=(3\times 0)-\{4\times(-1)\}=4$$

$$M_{31}=\begin{bmatrix}4 & 2\\ 8 & 4\end{bmatrix} \qquad \det(M_{31})=(4\times 4)-(2\times 8)=0$$

$$M_{32}=\begin{bmatrix}3 & 2\\ 6 & 4\end{bmatrix} \qquad \det(M_{32})=(3\times 4)-(2\times 6)=0$$

$$M_{33}=\begin{bmatrix}3 & 4\\ 6 & 8\end{bmatrix} \qquad \det(M_{33})=(3\times 8)-(4\times 6)=0$$

행렬 F의 소행렬 및 소행렬식

$$M_{11}=\begin{bmatrix}-1 & 3\\ -1 & 3\end{bmatrix} \qquad \det(M_{11})=\{(-1)\times 3\}-\{(-1)\times 3\}=0$$

$$M_{12}=\begin{bmatrix}2 & 3\\ 7 & 3\end{bmatrix} \qquad \det(M_{12})=(2\times 3)-(3\times 7)=-15$$

$$M_{13}=\begin{bmatrix}2 & -1\\ 7 & -1\end{bmatrix} \qquad \det(M_{13})=\{2\times(-1)\}-\{(-1)\times 7\}=5$$

$$M_{21}=\begin{bmatrix}0 & 1\\ -1 & 3\end{bmatrix} \qquad \det(M_{21})=(0\times 3)-\{1\times(-1)\}=1$$

$$M_{22}=\begin{bmatrix}-9 & 1\\ 7 & 3\end{bmatrix} \qquad \det(M_{22})=\{(-9)\times 3\}-(1\times 7)=-34$$

$$M_{23}=\begin{bmatrix}-9 & 0\\ 7 & -1\end{bmatrix} \qquad \det(M_{23})=\{(-9)\times(-1)\}-(0\times 7)=9$$

$$M_{31}=\begin{bmatrix}0 & 1\\ -1 & 3\end{bmatrix} \qquad \det(M_{31})=(0\times 3)-\{1\times(-1)\}=1$$

$$M_{32}=\begin{bmatrix}-9 & 1\\ 2 & 3\end{bmatrix} \qquad \det(M_{32})=\{(-9)\times 3\}-(1\times 2)=-29$$

$$M_{33}=\begin{bmatrix}-9 & 0\\ 2 & -1\end{bmatrix} \qquad \det(M_{33})=\{(-9)\times(-1)\}-(0\times 2)=9$$

행렬 G의 소행렬 및 소행렬식

$M_{11} = \begin{bmatrix} -4 & 5 \\ -3 & 3 \end{bmatrix}$ $\qquad$ $\det(M_{11}) = \{(-4)\times 3\} - \{5\times(-3)\} = 3$

$M_{12} = \begin{bmatrix} 6 & 5 \\ 4 & 3 \end{bmatrix}$ $\qquad$ $\det(M_{12}) = (6\times 3) - (5\times 4) = -2$

$M_{13} = \begin{bmatrix} 6 & -4 \\ 4 & -3 \end{bmatrix}$ $\qquad$ $\det(M_{13}) = \{6\times(-3)\} - \{(-4)\times 4\} = -2$

$M_{21} = \begin{bmatrix} -2 & 0 \\ -3 & 3 \end{bmatrix}$ $\qquad$ $\det(M_{21}) = \{(-2)\times 3\} - \{0\times(-3)\} = -6$

$M_{22} = \begin{bmatrix} 3 & 0 \\ 4 & 3 \end{bmatrix}$ $\qquad$ $\det(M_{22}) = (3\times 3) - (0\times 4) = 9$

$M_{23} = \begin{bmatrix} 3 & -2 \\ 4 & -3 \end{bmatrix}$ $\qquad$ $\det(M_{23}) = \{3\times(-3)\} - \{(-2)\times 4\} = -1$

$M_{31} = \begin{bmatrix} -2 & 0 \\ -4 & 5 \end{bmatrix}$ $\qquad$ $\det(M_{31}) = (-2)\times 5\} - \{0\times(-4)\} = -10$

$M_{32} = \begin{bmatrix} 3 & 0 \\ 6 & 5 \end{bmatrix}$ $\qquad$ $\det(M_{32}) = (3\times 5) - (0\times 6) = 15$

$M_{33} = \begin{bmatrix} 3 & -2 \\ 6 & -4 \end{bmatrix}$ $\qquad$ $\det(M_{33}) = \{3\times(-4)\} - \{(-2)\times 6\} = 0$

(b)

행렬 A의 여인수행렬 및 행렬식

$A_{11} = (-1)^{1+1}\det(M_{11}) = -7,$ $\qquad$ $A_{12} = (-1)^{1+2}\det(M_{12}) = 3,$

$A_{13} = (-1)^{1+3}\det(M_{13}) = -12,$ $\qquad$ $A_{21} = (-1)^{2+1}\det(M_{21}) = -5,$

$A_{22} = (-1)^{2+2}\det(M_{22}) = 2,$ $\qquad$ $A_{23} = (-1)^{2+3}\det(M_{23}) = -8,$

$A_{31} = (-1)^{3+1}\det(M_{31}) = 8,$ $\qquad$ $A_{32} = (-1)^{3+2}\det(M_{32}) = -3,$

$A_{33} = (-1)^{3+3}\det(M_{33}) = 13$

$$[A_{ij}] = \begin{bmatrix} -7 & 3 & -12 \\ -5 & 2 & -8 \\ 8 & -3 & 13 \end{bmatrix}$$

1열 선택 : $\det(A) = \{(-7)\times 2\} + \{(-5)\times(-3)\} + (8\times 0) = 1$

행렬 B의 여인수행렬 및 행렬식

$A_{11} = (-1)^{1+1}\det(M_{11}) = 69,$ $\qquad$ $A_{12} = (-1)^{1+2}\det(M_{12}) = -12,$

$A_{13} = (-1)^{1+3}\det(M_{13}) = -15,$ $\qquad$ $A_{21} = (-1)^{2+1}\det(M_{21}) = 0,$

$A_{22} = (-1)^{2+2}\det(M_{22}) = 0,$ $\qquad$ $A_{23} = (-1)^{2+3}\det(M_{23}) = 0,$

$A_{31} = (-1)^{3+1}\det(M_{31}) = -23,$ $\qquad$ $A_{32} = (-1)^{3+2}\det(M_{32}) = 4,$

$A_{33} = (-1)^{3+3}\det(M_{33}) = 5$

$$[A_{ij}] = \begin{bmatrix} 69 & -12 & -15 \\ 0 & 0 & 0 \\ -23 & 4 & 5 \end{bmatrix}$$

2행 선택 : $\det(B) = (0\times 0) + (0\times 5) + \{0\times(-4)\} = 0$

행렬 C의 여인수행렬 및 행렬식

$A_{11}=(-1)^{1+1}\det(M_{11})=10$, $A_{12}=(-1)^{1+2}\det(M_{12})=0$,

$A_{13}=(-1)^{1+3}\det(M_{13})=0$, $A_{21}=(-1)^{2+1}\det(M_{21})=45$,

$A_{22}=(-1)^{2+2}\det(M_{22})=0$, $A_{23}=(-1)^{2+3}\det(M_{23})=0$,

$A_{31}=(-1)^{3+1}\det(M_{31})=-30$, $A_{32}=(-1)^{3+2}\det(M_{32})=0$,

$A_{33}=(-1)^{3+3}\det(M_{33})=0$

$$[A_{ij}]=\begin{bmatrix} 10 & 0 & 0 \\ 45 & 0 & 0 \\ -30 & 0 & 0 \end{bmatrix}$$

1열 선택 : $\det(C)=(10\times 0)+(45\times 0)+(30\times 0)=0$

행렬 D의 여인수행렬 및 행렬식

$A_{11}=(-1)^{1+1}\det(M_{11})=-3$, $A_{12}=(-1)^{1+2}\det(M_{12})=-7$,

$A_{13}=(-1)^{1+3}\det(M_{13})=-1$, $A_{21}=(-1)^{2+1}\det(M_{21})=6$,

$A_{22}=(-1)^{2+2}\det(M_{22})=7$, $A_{23}=(-1)^{2+3}\det(M_{23})=-5$,

$A_{31}=(-1)^{3+1}\det(M_{31})=24$, $A_{32}=(-1)^{3+2}\det(M_{32})=14$,

$A_{33}=(-1)^{3+3}\det(M_{33})=-13$

$$[A_{ij}]=\begin{bmatrix} -3 & -7 & -1 \\ 6 & 7 & -5 \\ 24 & 14 & -13 \end{bmatrix}$$

1행 선택 : $\det(D)=\{(-3)\times 1\}+\{(-7)\times 2\}+\{(-1)\times 4\}=-21$

행렬 E의 여인수행렬 및 행렬식

$A_{11}=(-1)^{1+1}\det(M_{11})=8$, $A_{12}=(-1)^{1+2}\det(M_{12})=-10$,

$A_{13}=(-1)^{1+3}\det(M_{13})=8$, $A_{21}=(-1)^{2+1}\det(M_{21})=-4$,

$A_{22}=(-1)^{2+2}\det(M_{22})=5$, $A_{23}=(-1)^{2+3}\det(M_{23})=-4$,

$A_{31}=(-1)^{3+1}\det(M_{31})=0$, $A_{32}=(-1)^{3+2}\det(M_{32})=0$,

$A_{33}=(-1)^{3+3}\det(M_{33})=0$

$$[A_{ij}]=\begin{bmatrix} 8 & -10 & 8 \\ -4 & 5 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

3행 선택 : $\det(E)=\{0\times(-1)\}+(0\times 0)+(0\times 1)=0$

행렬 F의 여인수행렬 및 행렬식

$A_{11}=(-1)^{1+1}\det(M_{11})=0$, $A_{12}=(-1)^{1+2}\det(M_{12})=15$,

$A_{13}=(-1)^{1+3}\det(M_{13})=5$, $A_{21}=(-1)^{2+1}\det(M_{21})=-1$,

$A_{22}=(-1)^{2+2}\det(M_{22})=-34$, $A_{23}=(-1)^{2+3}\det(M_{23})=-9$,

$A_{31}=(-1)^{3+1}\det(M_{31})=1$, $A_{32}=(-1)^{3+2}\det(M_{32})=29$,

$A_{33}=(-1)^{3+3}\det(M_{33})=9$

$$[A_{ij}]=\begin{bmatrix} 0 & 15 & 5 \\ -1 & -34 & -9 \\ 1 & 29 & 9 \end{bmatrix}$$

1행 선택 : $\det(F)=\{0\times(-9)\}+(15\times 0)+(5\times 1)=5$

행렬 G의 여인수행렬 및 행렬식

$A_{11}=(-1)^{1+1}\det(M_{11})=3$, $\quad A_{12}=(-1)^{1+2}\det(M_{12})=2$,

$A_{13}=(-1)^{1+3}\det(M_{13})=-2$, $\quad A_{21}=(-1)^{2+1}\det(M_{21})=6$,

$A_{22}=(-1)^{2+2}\det(M_{22})=9$, $\quad A_{23}=(-1)^{2+3}\det(M_{23})=1$,

$A_{31}=(-1)^{3+1}\det(M_{31})=-10$, $\quad A_{32}=(-1)^{3+2}\det(M_{32})=-15$,

$A_{33}=(-1)^{3+3}\det(M_{33})=0$

$$[A_{ij}]=\begin{bmatrix}3 & 2 & -2\\ 6 & 9 & 1\\ -10 & -15 & 0\end{bmatrix}$$

3열 선택 : $\det(G)=\{(-2)\times 0\}+(1\times 5)+(0\times 3)=5$

(c)

행렬 A의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}-7 & -5 & 8\\ 3 & 2 & -3\\ -12 & -8 & 13\end{bmatrix}$

행렬 B의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}69 & 0 & -23\\ -12 & 0 & 4\\ -15 & 0 & 5\end{bmatrix}$

행렬 C의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}10 & 45 & =30\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

행렬 D의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}-3 & 6 & 24\\ -7 & 7 & 14\\ -1 & -5 & -13\end{bmatrix}$

행렬 E의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}8 & -4 & 0\\ -10 & 5 & 0\\ 8 & -4 & 0\end{bmatrix}$

행렬 F의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}0 & -1 & 1\\ 15 & -34 & 29\\ 5 & -9 & 9\end{bmatrix}$

행렬 G의 수반행렬 : $[A_{ij}]^T=\begin{bmatrix}3 & 6 & -10\\ 2 & 9 & -15\\ -2 & 1 & 0\end{bmatrix}$

(d) 가역행렬 : 행렬 A, D, F, G

특이행렬 : B, C, E

(e)

행렬 A의 역행렬 : $A^{-1}=\begin{bmatrix}-7 & -5 & 8\\ 3 & 2 & -3\\ -12 & -8 & 13\end{bmatrix}$

행렬 D의 역행렬 : $D^{-1}=-\frac{1}{21}\begin{bmatrix}-3 & 6 & 24\\ -7 & 7 & 14\\ -1 & -5 & -13\end{bmatrix}$

행렬 F의 역행렬 : $F^{-1}=\frac{1}{5}\begin{bmatrix}0 & -1 & 1\\ 15 & -34 & 29\\ 5 & -9 & 9\end{bmatrix}$

행렬 G의 역행렬 : $G^{-1}=\frac{1}{5}\begin{bmatrix}3 & 6 & -10\\ 2 & 9 & -15\\ -2 & 1 & 0\end{bmatrix}$

(f)

행렬 A의 역행렬

1) 1행 = 1행 $\times \frac{1}{2}$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ -3 & 5 & 3 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$ 1행 = 1행 $\times \frac{1}{2}$

2) 2행 = 1행 $\times 3$ + 2행

과정

1행 $\times 3$ $$\left[\begin{array}{ccc|ccc} 3 & \frac{3}{2} & -\frac{3}{2} & \frac{3}{2} & 0 & 0 \\ -3 & 5 & 3 & 0 & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{13}{2} & \frac{3}{2} & \frac{3}{2} & 1 & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$ 1행 : 원래대로 작성

2행 = 1행 $\times 3$ + 2행

3) 2행 = 2행 $\times \frac{2}{13}$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$ 2행 = 2행 $\times \frac{2}{13}$

4) 1행 = 2행 $\times \left(-\frac{1}{2}\right)$ + 1행

과정

2행 $\times \left(-\frac{1}{2}\right)$ $$\left[\begin{array}{ccc|ccc} 1 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{3}{26} & -\frac{3}{26} & -\frac{2}{26} & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{26} & \frac{10}{26} & -\frac{2}{26} & 0 \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$ 1행 = 2행 $\times \left(-\frac{1}{2}\right)$ + 1행

2행 : 원래대로 작성

5) 3행 = 2행 $\times -4$ + 3행

과정

$$2행 \times (-4) \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{26} & \frac{10}{26} & -\frac{2}{26} & 0 \\ 0 & -4 & -\frac{12}{13} & -\frac{12}{13} & -\frac{8}{13} & 0 \\ 0 & 4 & 1 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{26} & \frac{10}{26} & -\frac{2}{26} & 0 \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 0 & \frac{1}{13} & -\frac{12}{13} & -\frac{8}{13} & 1 \end{array}\right] \begin{array}{l} \\ 2행 : 원래대로 작성 \\ 3행 = 2행 \times (-4) + 3행 \end{array}$$

6) 3행 = 3행 $\times 13$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{26} & \frac{10}{26} & -\frac{2}{26} & 0 \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 0 & 1 & -12 & -8 & 13 \end{array}\right] \quad 3행 = 3행 \times 13$$

7) 1행 = 3행 $\times \frac{16}{26}$ + 1행

과정

$$3행 \times \frac{16}{26} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{26} & \frac{10}{26} & -\frac{2}{26} & 0 \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 0 & \frac{16}{26} & -\frac{192}{26} & -\frac{128}{26} & \frac{208}{26} \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{182}{26} & -\frac{130}{26} & \frac{208}{26} \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 0 & 1 & -12 & -8 & 13 \end{array}\right] \begin{array}{l} 1행 = 3행 \times \frac{16}{26} + 1행 \\ \\ 3행 : 원래대로 작성 \end{array}$$

8) 2행 = 3행 $\times\left(-\frac{3}{13}\right)$ + 2행

과정

$$3\text{행}\ \times\left(-\frac{3}{13}\right)\quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{182}{26} & -\frac{130}{26} & \frac{208}{26} \\ 0 & 1 & \frac{3}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 0 & -\frac{3}{13} & \frac{36}{13} & \frac{24}{13} & -\frac{39}{13} \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{182}{26} & -\frac{130}{26} & \frac{208}{26} \\ 0 & 1 & 0 & \frac{39}{13} & \frac{26}{13} & -\frac{39}{13} \\ 0 & 0 & 1 & -12 & -8 & 13 \end{array}\right]$$

2행 = 3행 $\times\left(-\frac{3}{13}\right)$ + 2행
3행 : 원래대로 작성

$$\therefore\ A^{-1} = \begin{bmatrix} -\frac{182}{26} & -\frac{130}{26} & \frac{208}{26} \\ \frac{39}{13} & \frac{26}{13} & -\frac{39}{13} \\ -12 & -8 & 13 \end{bmatrix} = \begin{bmatrix} -7 & -5 & 8 \\ 3 & 2 & -3 \\ -12 & -8 & 13 \end{bmatrix}$$

행렬 D의 역행렬

1) 2행 = 1행 $\times(-5)$ + 2행

과정

$$1\text{행}\ \times(-5)\quad \left[\begin{array}{ccc|ccc} -5 & -10 & -20 & -5 & 0 & 0 \\ 5 & -3 & 6 & 0 & 1 & 0 \\ -2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & -13 & -14 & -5 & 1 & 0 \\ -2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

1행 : 원래대로 작성
2행 = 1행 $\times(-5)$ + 2행

2) 3행 = 1행 $\times 2$ + 3행

과정

$$1\text{행}\ \times 2\quad \left[\begin{array}{ccc|ccc} 2 & 4 & 8 & 2 & 0 & 0 \\ 0 & -13 & -14 & -5 & 1 & 0 \\ -2 & 1 & -1 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & -13 & -14 & -5 & 1 & 0 \\ 0 & 5 & 7 & 2 & 0 & 1 \end{array}\right]$$

1행 : 원래대로 작성

3행 = 1행 $\times 2$ + 3행

3) 2행 = 2행 $\times\left(-\frac{1}{13}\right)$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 5 & 7 & 2 & 0 & 1 \end{array}\right]$$

2행 = 2행 $\times\left(-\frac{1}{13}\right)$

4) 1행 = 2행 $\times(-2)$ + 1행

과정

$$2행\ \times(-2)\quad \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 1 & 0 & 0 \\ 0 & -2 & -\frac{28}{13} & -\frac{10}{13} & \frac{2}{13} & 0 \\ 0 & 5 & 7 & 2 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 5 & 7 & 2 & 0 & 1 \end{array}\right]$$

1행 = 2행 $\times(-2)$ + 1행

2행 : 원래대로 작성

5) 3행 = 2행 $\times(-5)$ + 3행

과정

$$2행\ \times(-5)\quad \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & -5 & -\frac{70}{13} & -\frac{25}{13} & \frac{5}{13} & 0 \\ 0 & 5 & 7 & 2 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 0 & \frac{21}{13} & \frac{1}{13} & \frac{5}{13} & 1 \end{array}\right]$$

2행 : 원래대로 작성

3행 = 2행 $\times(-5)$ + 3행

6) 3행 = 3행 $\times\frac{13}{21}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 0 & 1 & \frac{1}{21} & \frac{5}{21} & \frac{13}{21} \end{array}\right]$$

3행 = 3행 $\times\frac{13}{21}$

7) 1행 = 3행 $\times\left(-\frac{24}{13}\right)$ + 1행

과정

$$3\text{행} \times\left(-\frac{24}{13}\right) \quad \left[\begin{array}{ccc|ccc} 1 & 0 & \frac{24}{13} & \frac{3}{13} & \frac{2}{13} & 0 \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 0 & -\frac{24}{13} & -\frac{24}{273} & -\frac{120}{273} & -\frac{312}{273} \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{39}{273} & -\frac{78}{273} & -\frac{312}{273} \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 0 & 1 & \frac{1}{21} & \frac{5}{21} & \frac{13}{21} \end{array}\right]$$

1행 = 3행 $\times\left(-\frac{24}{13}\right)$ + 1행

3행 : 원래대로 작성

8) 2행 = 3행 $\times\left(-\frac{14}{13}\right)$ + 2행

과정

$$3\text{행} \times\left(-\frac{14}{13}\right) \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{39}{273} & -\frac{78}{273} & -\frac{312}{273} \\ 0 & 1 & \frac{14}{13} & \frac{5}{13} & -\frac{1}{13} & 0 \\ 0 & 0 & -\frac{14}{13} & -\frac{14}{273} & -\frac{70}{273} & -\frac{182}{273} \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{39}{273} & -\frac{78}{273} & -\frac{312}{273} \\ 0 & 1 & 0 & \frac{91}{273} & -\frac{91}{273} & -\frac{182}{273} \\ 0 & 0 & 1 & \frac{1}{21} & \frac{5}{21} & \frac{13}{21} \end{array}\right]$$

2행 = 3행 $\times\left(-\frac{14}{13}\right)$ + 2행

3행 : 원래대로 작성

$$\therefore\ D^{-1}=\begin{bmatrix} \frac{39}{273} & -\frac{78}{273} & -\frac{312}{273} \\ \frac{91}{273} & -\frac{91}{273} & -\frac{182}{273} \\ \frac{1}{21} & \frac{5}{21} & \frac{13}{21} \end{bmatrix}=-\frac{1}{21}\begin{bmatrix} -3 & 6 & 24 \\ -7 & 7 & 14 \\ -1 & -5 & -13 \end{bmatrix}$$

행렬 F의 역행렬

1) 1행 = 1행 $\times\left(-\frac{1}{9}\right)$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 7 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

1행 = 1행 $\times\left(-\frac{1}{9}\right)$

2) 2행 = 1행 ×(−2) + 2행

과정

$$1\text{행} \times(-2) \quad \left[\begin{array}{ccc|ccc} -2 & 0 & \frac{2}{9} & \frac{2}{9} & 0 & 0 \\ 2 & -1 & 3 & 0 & 1 & 0 \\ 7 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & -1 & \frac{29}{9} & \frac{2}{9} & 1 & 0 \\ 7 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

1행 : 원래대로 작성
2행 = 1행 ×(−2) + 2행

3) 3행 = 1행 ×(−7) + 3행

과정

$$1\text{행} \times(-7) \quad \left[\begin{array}{ccc|ccc} -7 & 0 & \frac{7}{9} & \frac{7}{9} & 0 & 0 \\ 0 & -1 & \frac{29}{9} & \frac{2}{9} & 1 & 0 \\ 7 & -1 & 3 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & -1 & \frac{29}{9} & \frac{2}{9} & 1 & 0 \\ 0 & -1 & \frac{34}{9} & \frac{7}{9} & 0 & 1 \end{array}\right]$$

1행 : 원래대로 작성

3행 = 1행 ×(−7) + 3행

4) 2행 = 2행 ×(−1)

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & -1 & \frac{34}{9} & \frac{7}{9} & 0 & 1 \end{array}\right]$$

2행 = 2행 ×(−1)

5) 3행 = 2행 × 1 + 3행

과정

$$2\text{행} \times 1 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & -1 & \frac{34}{9} & \frac{7}{9} & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & 0 & \frac{5}{9} & \frac{5}{9} & -1 & 1 \end{array}\right]$$

2행 : 원래대로 작성
3행 = 2행 × 1 + 3행

6) 3행 = 3행 $\times \frac{9}{5}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & 0 & 1 & 1 & -\frac{9}{5} & \frac{9}{5} \end{array}\right] \quad \text{3행 = 3행} \times \frac{9}{5}$$

7) 1행 = 3행 $\times \frac{1}{9}$ + 1행

과정

$$\text{3행} \times \frac{1}{9} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{9} & -\frac{1}{9} & 0 & 0 \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & 0 & \frac{1}{9} & \frac{1}{9} & -\frac{1}{5} & \frac{1}{5} \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & 0 & 1 & 1 & -\frac{9}{5} & \frac{9}{5} \end{array}\right] \quad \begin{array}{l} \text{1행 = 3행} \times \frac{1}{9} + \text{1행} \\ \\ \text{3행 : 원래대로 작성} \end{array}$$

8) 2행 = 3행 $\times \frac{29}{9}$ + 2행

과정

$$\text{3행} \times \frac{29}{9} \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{29}{9} & -\frac{2}{9} & -1 & 0 \\ 0 & 0 & \frac{29}{9} & \frac{29}{9} & -\frac{29}{5} & \frac{29}{5} \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -\frac{1}{5} & \frac{1}{5} \\ 0 & 1 & 0 & \frac{27}{9} & -\frac{34}{5} & \frac{29}{5} \\ 0 & 0 & 1 & 1 & -\frac{9}{5} & \frac{9}{5} \end{array}\right] \quad \begin{array}{l} \text{2행 = 3행} \times \frac{29}{9} + \text{2행} \\ \text{3행 : 원래대로 작성} \end{array}$$

$$\therefore \; F^{-1} = \begin{bmatrix} 0 & -\frac{1}{5} & \frac{1}{5} \\ \frac{27}{9} & -\frac{34}{5} & \frac{29}{5} \\ 1 & -\frac{9}{5} & \frac{9}{5} \end{bmatrix} F^{-1} = \frac{1}{5}\begin{bmatrix} 0 & -1 & 1 \\ 15 & -34 & 29 \\ 5 & -9 & 9 \end{bmatrix}$$

행렬 G의 역행렬

1) 1행 = 1행 $\times \frac{1}{3}$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 6 & -4 & 5 & 0 & 1 & 0 \\ 4 & -3 & 3 & 0 & 0 & 1 \end{array}\right]$$ 1행 = 1행 $\times \frac{1}{3}$

2) 2행 = 1행 $\times(-6)$ + 2행

과정

1행 $\times(-6)$ $$\left[\begin{array}{ccc|ccc} -6 & 4 & 0 & -2 & 0 & 0 \\ 6 & -4 & 5 & 0 & 1 & 0 \\ 4 & -3 & 3 & 0 & 0 & 1 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 5 & -2 & 1 & 0 \\ 4 & -3 & 3 & 0 & 0 & 1 \end{array}\right]$$

1행 : 원래대로 작성
2행 = 1행 $\times(-6)$ + 2행

3) 2행과 3행 자리바꿈

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 4 & -3 & 3 & 0 & 0 & 1 \\ 0 & 0 & 5 & -2 & 1 & 0 \end{array}\right]$$

4) 2행 = 1행 $\times(-4)$ + 2행

과정

1행 $\times(-4)$ $$\left[\begin{array}{ccc|ccc} -4 & \frac{8}{3} & 0 & -\frac{4}{3} & 0 & 0 \\ 4 & -3 & 3 & 0 & 0 & 1 \\ 0 & 0 & 5 & -2 & 1 & 0 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 3 & -\frac{4}{3} & 0 & 1 \\ 0 & 0 & 5 & -2 & 1 & 0 \end{array}\right]$$

1행 : 원래대로 작성

2행 = 1행 $\times(-4)$ + 2행

5) 2행 = 2행 $\times(-3)$

$$\left[\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & 1 & -9 & 4 & 0 & -3 \\ 0 & 0 & 5 & -2 & 1 & 0 \end{array}\right]$$ 2행 = 2행 $\times(-3)$

6) 1행 = 2행 $\times \frac{2}{3}$ + 1행

과정

$$2행\ \times \frac{2}{3} \quad \left[\begin{array}{ccc|ccc} 1 & -\frac{2}{3} & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{2}{3} & -6 & \frac{8}{3} & 0 & -2 \\ 0 & 0 & 5 & -2 & 1 & 0 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -6 & 3 & 0 & -2 \\ 0 & 1 & -9 & 4 & 0 & -3 \\ 0 & 0 & 5 & -2 & 1 & 0 \end{array}\right]$$

1행 = 2행 $\times \frac{2}{3}$ + 1행

2행 : 원래대로 작성

7) 3행 = 3행 $\times \frac{1}{5}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -6 & 3 & 0 & -2 \\ 0 & 1 & -9 & 4 & 0 & -3 \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{1}{5} & 0 \end{array}\right]$$

3행 = 3행 $\times \frac{1}{5}$

8) 1행 = 3행 $\times$ 6 + 1행

과정

$$3행\ \times 6 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & -6 & 3 & 0 & -2 \\ 0 & 1 & -9 & 4 & 0 & -3 \\ 0 & 0 & 6 & -\frac{12}{5} & \frac{6}{5} & 0 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & \frac{6}{5} & -2 \\ 0 & 1 & -9 & 4 & 0 & -3 \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{1}{5} & 0 \end{array}\right]$$

1행 = 3행 $\times$ 6 + 1행

3행 : 원래대로 작성

9) 2행 = 3행 $\times$ 9 + 2행

과정

$$3행\ \times 9 \quad \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & \frac{6}{5} & -2 \\ 0 & 1 & -9 & 4 & 0 & -3 \\ 0 & 0 & 9 & -\frac{18}{5} & \frac{9}{5} & 0 \end{array}\right]$$

결과

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{5} & \frac{6}{5} & -2 \\ 0 & 1 & 0 & \frac{2}{5} & \frac{9}{5} & -3 \\ 0 & 0 & 1 & -\frac{2}{5} & \frac{1}{5} & 0 \end{array}\right]$$

2행 = 3행 $\times$ 9 + 2행

3행 : 원래대로 작성

$$\therefore \ G^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{6}{5} & -2 \\ \frac{2}{5} & \frac{9}{5} & -3 \\ -\frac{2}{5} & \frac{1}{5} & 0 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & 6 & -10 \\ 2 & 9 & -15 \\ -2 & 1 & 0 \end{bmatrix}$$

14.

(a)

행렬 A의 소행렬과 행렬식

소행렬	소행렬식
$M_{11} = \begin{bmatrix} -2 & 5 & -1 \\ -4 & 4 & 0 \\ 0 & -5 & 2 \end{bmatrix}$	4
$M_{12} = \begin{bmatrix} 0 & 5 & -1 \\ 3 & 4 & 0 \\ 1 & -5 & 2 \end{bmatrix}$	-11
$M_{13} = \begin{bmatrix} 0 & -2 & -1 \\ 3 & -4 & 0 \\ 1 & 0 & 2 \end{bmatrix}$	8
$M_{14} = \begin{bmatrix} 0 & -2 & 5 \\ 3 & -4 & 4 \\ 1 & 0 & -5 \end{bmatrix}$	-18
$M_{21} = \begin{bmatrix} 3 & 0 & 2 \\ -4 & 4 & 0 \\ 0 & -5 & 2 \end{bmatrix}$	64
$M_{22} = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 0 \\ 1 & -5 & 2 \end{bmatrix}$	-30
$M_{23} = \begin{bmatrix} 1 & 3 & 2 \\ 3 & -4 & 0 \\ 1 & 0 & 2 \end{bmatrix}$	-18
$M_{24} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & -4 & 4 \\ 1 & 0 & -5 \end{bmatrix}$	77
$M_{31} = \begin{bmatrix} 3 & 0 & 2 \\ -2 & 5 & -1 \\ 0 & -5 & 2 \end{bmatrix}$	35
$M_{32} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -1 \\ 1 & -5 & 2 \end{bmatrix}$	-5
$M_{33} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$	-3
$M_{34} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & 5 \\ 1 & 0 & -5 \end{bmatrix}$	25
$M_{41} = \begin{bmatrix} 3 & 0 & 2 \\ -2 & 5 & -1 \\ -4 & 4 & 0 \end{bmatrix}$	36
$M_{42} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -1 \\ 3 & 4 & 0 \end{bmatrix}$	-26

$M_{43} = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -2 & -1 \\ 3 & -4 & 0 \end{bmatrix}$	-1
$M_{44} = \begin{bmatrix} 1 & 3 & 0 \\ 0 & -2 & 5 \\ 3 & -4 & 4 \end{bmatrix}$	57

행렬 B의 소행렬과 행렬식

소행렬	소행렬식
$M_{11} = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 2 & -3 \\ 0 & -3 & -1 \end{bmatrix}$	-9
$M_{12} = \begin{bmatrix} 0 & 4 & 1 \\ 4 & 2 & -3 \\ -1 & -3 & -1 \end{bmatrix}$	18
$M_{13} = \begin{bmatrix} 0 & 1 & 1 \\ 4 & 2 & -3 \\ -1 & 0 & -1 \end{bmatrix}$	9
$M_{14} = \begin{bmatrix} 0 & 1 & 4 \\ 4 & 2 & 2 \\ -1 & 0 & -3 \end{bmatrix}$	18
$M_{21} = \begin{bmatrix} -1 & 5 & 2 \\ 2 & 2 & -3 \\ 0 & -3 & -1 \end{bmatrix}$	9
$M_{22} = \begin{bmatrix} 3 & 5 & 2 \\ 4 & 2 & -3 \\ -1 & -3 & -1 \end{bmatrix}$	-18
$M_{23} = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & -3 \\ -1 & 0 & -1 \end{bmatrix}$	-9
$M_{24} = \begin{bmatrix} 3 & -1 & 5 \\ 4 & 2 & 2 \\ -1 & 0 & -3 \end{bmatrix}$	-18
$M_{31} = \begin{bmatrix} -1 & 5 & 2 \\ 1 & 4 & 1 \\ 0 & -3 & -1 \end{bmatrix}$	0
$M_{32} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 4 & 1 \\ -1 & -3 & -1 \end{bmatrix}$	0
$M_{33} = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix}$	0
$M_{34} = \begin{bmatrix} 3 & -1 & 5 \\ 0 & 1 & 4 \\ -1 & 0 & -3 \end{bmatrix}$	0
$M_{41} = \begin{bmatrix} -1 & 5 & 2 \\ 1 & 4 & 1 \\ 2 & 2 & -3 \end{bmatrix}$	27
$M_{42} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 4 & 1 \\ 4 & 2 & -3 \end{bmatrix}$	-54

$M_{43} = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & 1 \\ 4 & 2 & -3 \end{bmatrix}$	-27
$M_{44} = \begin{bmatrix} 3 & -1 & 5 \\ 0 & 1 & 4 \\ 4 & 2 & 2 \end{bmatrix}$	-54

행렬 C의 소행렬과 행렬식

소행렬	소행렬식
$M_{11} = \begin{bmatrix} 0 & -5 & 3 \\ -2 & -3 & 1 \\ 1 & -4 & 3 \end{bmatrix}$	-2
$M_{12} = \begin{bmatrix} 2 & -5 & 3 \\ 4 & -3 & 1 \\ 2 & -4 & 3 \end{bmatrix}$	10
$M_{13} = \begin{bmatrix} 2 & 0 & 3 \\ 4 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$	10
$M_{14} = \begin{bmatrix} 2 & 0 & -5 \\ 4 & -2 & -3 \\ 2 & 1 & -4 \end{bmatrix}$	-18
$M_{21} = \begin{bmatrix} -1 & -4 & 2 \\ -2 & -3 & 1 \\ 1 & -4 & 3 \end{bmatrix}$	-1
$M_{22} = \begin{bmatrix} 1 & -4 & 2 \\ 4 & -3 & 1 \\ 2 & -4 & 3 \end{bmatrix}$	15
$M_{23} = \begin{bmatrix} 1 & -1 & 2 \\ 4 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$	19
$M_{24} = \begin{bmatrix} 1 & -1 & -4 \\ 4 & -2 & -3 \\ 2 & 1 & -4 \end{bmatrix}$	-31
$M_{31} = \begin{bmatrix} -1 & -4 & 2 \\ 0 & -5 & 3 \\ 1 & -4 & 3 \end{bmatrix}$	1
$M_{32} = \begin{bmatrix} 1 & -4 & 2 \\ 2 & -5 & 3 \\ 2 & -4 & 3 \end{bmatrix}$	1
$M_{33} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 2 & 1 & 3 \end{bmatrix}$	1
$M_{34} = \begin{bmatrix} 1 & -1 & -4 \\ 2 & 0 & -5 \\ 2 & 1 & -4 \end{bmatrix}$	-1
$M_{41} = \begin{bmatrix} -1 & -4 & 2 \\ 0 & -5 & 3 \\ -2 & -3 & 1 \end{bmatrix}$	0
$M_{42} = \begin{bmatrix} 1 & -4 & 2 \\ 2 & -5 & 3 \\ 4 & -3 & 1 \end{bmatrix}$	-8
$M_{43} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 0 & 3 \\ 4 & -2 & 1 \end{bmatrix}$	-12

$M_{44}=\begin{bmatrix}1 & -1 & -4\\ 2 & 0 & -5\\ 4 & -2 & -3\end{bmatrix}$	20

행렬 D의 소행렬과 행렬식

소행렬	소행렬식
$M_{11}=\begin{bmatrix}4 & 4 & -5\\ 2 & -1 & -2\\ 1 & 0 & 3\end{bmatrix}$	-49
$M_{12}=\begin{bmatrix}6 & 4 & -5\\ 1 & -1 & -2\\ -2 & 0 & 3\end{bmatrix}$	-4
$M_{13}=\begin{bmatrix}6 & 4 & -5\\ 1 & 2 & -2\\ -2 & 1 & 3\end{bmatrix}$	27
$M_{14}=\begin{bmatrix}6 & 4 & 4\\ 1 & 2 & -1\\ -2 & 1 & 0\end{bmatrix}$	34
$M_{21}=\begin{bmatrix}9 & 0 & -6\\ 2 & -1 & -2\\ 1 & 0 & 3\end{bmatrix}$	-33
$M_{22}=\begin{bmatrix}5 & 0 & -6\\ 1 & -1 & -2\\ -2 & 0 & 3\end{bmatrix}$	-3
$M_{23}=\begin{bmatrix}5 & 9 & -6\\ 1 & 2 & -2\\ -2 & 1 & 3\end{bmatrix}$	19
$M_{24}=\begin{bmatrix}5 & 9 & 0\\ 1 & 2 & -1\\ -2 & 1 & 0\end{bmatrix}$	23
$M_{31}=\begin{bmatrix}9 & 0 & -6\\ 4 & 4 & -5\\ 1 & 0 & 3\end{bmatrix}$	132
$M_{32}=\begin{bmatrix}5 & 0 & -6\\ 6 & 4 & -5\\ -2 & 0 & 3\end{bmatrix}$	12
$M_{33}=\begin{bmatrix}5 & 9 & -6\\ 6 & 4 & -5\\ -2 & 1 & 3\end{bmatrix}$	-71
$M_{34}=\begin{bmatrix}5 & 9 & 0\\ 6 & 4 & 4\\ -2 & 1 & 0\end{bmatrix}$	-92
$M_{41}=\begin{bmatrix}9 & 0 & -6\\ 4 & 4 & -5\\ 2 & -1 & -2\end{bmatrix}$	-45
$M_{42}=\begin{bmatrix}5 & 0 & -6\\ 6 & 4 & -5\\ 1 & -1 & -2\end{bmatrix}$	-5
$M_{43}=\begin{bmatrix}5 & 9 & -6\\ 6 & 4 & -5\\ 1 & 2 & -2\end{bmatrix}$	25
$M_{44}=\begin{bmatrix}5 & 9 & 0\\ 6 & 4 & 4\\ 1 & 2 & -1\end{bmatrix}$	30

(b)

행렬 A에 대한 여인수행렬

$$[A_{ij}] = \begin{bmatrix} 4 & 11 & 8 & 18 \\ -64 & -30 & 18 & 77 \\ 35 & 5 & -3 & -25 \\ -36 & -26 & 1 & 57 \end{bmatrix}$$

행렬 B에 대한 여인수행렬

$$[A_{ij}] = \begin{bmatrix} -9 & -18 & 9 & -18 \\ -9 & -18 & 9 & -18 \\ 0 & 0 & 0 & 0 \\ -27 & -54 & 27 & -54 \end{bmatrix}$$

행렬 C에 대한 여인수행렬

$$[A_{ij}] = \begin{bmatrix} -2 & -10 & 10 & 18 \\ 1 & 15 & -19 & -31 \\ 1 & -1 & 1 & 1 \\ 0 & -8 & 12 & 20 \end{bmatrix}$$

행렬 D에 대한 여인수행렬

$$[A_{ij}] = \begin{bmatrix} -49 & 4 & 27 & -34 \\ 33 & -3 & -19 & 23 \\ 132 & -12 & -71 & 92 \\ 45 & -5 & -25 & 30 \end{bmatrix}$$

(c)

행렬 A에 대한 수반행렬

$$[A_{ij}]^T = \begin{bmatrix} 4 & -64 & 35 & -36 \\ 11 & -30 & 5 & -26 \\ 8 & 18 & -3 & 1 \\ 18 & 77 & -25 & 57 \end{bmatrix}$$

행렬 B에 대한 수반행렬

$$[A_{ij}]^T = \begin{bmatrix} -9 & -9 & 0 & -27 \\ -18 & -18 & 0 & -54 \\ 9 & 9 & 0 & 27 \\ -18 & -18 & 0 & -54 \end{bmatrix}$$

행렬 C에 대한 수반행렬

$$[A_{ij}]^T = \begin{bmatrix} -2 & 1 & 1 & 0 \\ -10 & 15 & -1 & -8 \\ 10 & -19 & 1 & 12 \\ 18 & -31 & 1 & 20 \end{bmatrix}$$

행렬 D에 대한 수반행렬

$$[A_{ij}]^T = \begin{bmatrix} -49 & 33 & 132 & 45 \\ 4 & -3 & -12 & -5 \\ 27 & -19 & -71 & -25 \\ -34 & 23 & 92 & 30 \end{bmatrix}$$

(d)

행렬 A의 행렬식 : 73

행렬 B의 행렬식 : 0

행렬 C의 행렬식 : 4

행렬 D의 행렬식 : -5

(e)

행렬 A의 역행렬 : $A^{-1}=\frac{1}{73}\begin{bmatrix} 4 & -64 & 35 & -36 \\ 11 & -30 & 5 & -26 \\ 8 & 18 & -3 & 1 \\ 18 & 77 & -25 & 57 \end{bmatrix}$

행렬 B의 역행렬 : 역행렬을 구할 수 없다.

행렬 C의 역행렬 : $C^{-1}=\frac{1}{4}\begin{bmatrix} -2 & 1 & 1 & 0 \\ -10 & 15 & -1 & -8 \\ 10 & -19 & 1 & 12 \\ 18 & -31 & 1 & 20 \end{bmatrix}$

행렬 D의 역행렬 : $D^{-1}=-\frac{1}{5}\begin{bmatrix} -49 & 33 & 132 & 45 \\ 4 & -3 & -12 & -5 \\ 27 & -19 & -71 & -25 \\ -34 & 23 & 92 & 30 \end{bmatrix}$

(f) 풀이 생략

15.

(1) 가우스 소거법 이용

(a) 첨가행렬 : $\left[\begin{array}{ccc|c} 3 & -5 & 1 & -3 \\ 3 & 2 & -1 & 0 \\ -1 & -1 & 1 & 1 \end{array}\right]$

① 1행과 3행의 자리바꿈

$$\left[\begin{array}{ccc|c} -1 & -1 & 1 & 1 \\ 3 & 2 & -1 & 0 \\ 3 & -5 & 1 & -3 \end{array}\right]$$

② 1행 = 1행 $\times(-1)$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 3 & 2 & -1 & 0 \\ 3 & -5 & 1 & -3 \end{array}\right]$$ 1행 = 1행 $\times(-1)$

③ 2행 = 1행 $\times(-3)+$ 2행

과정 결과

1행 $\times(-3)$ $\left[\begin{array}{ccc|c} -3 & -3 & 3 & 3 \\ 3 & 2 & -1 & 0 \\ 3 & -5 & 1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & -1 & 2 & 3 \\ 3 & -5 & 1 & -3 \end{array}\right]$ 1행 : 원래대로 작성

2행 = 1행 $\times(-3)+$ 2행

④ 3행 = 1행 $\times(-3)+$ 3행

	과정		결과	
1행 $\times(-3)$	$\left[\begin{array}{ccc\|c} -3 & -3 & 3 & 3 \\ 0 & -1 & 2 & 3 \\ 3 & -5 & 1 & -3 \end{array}\right]$	$\Rightarrow$	$\left[\begin{array}{ccc\|c} 1 & 1 & -1 & -1 \\ 0 & -1 & 2 & 3 \\ 0 & -8 & 4 & 0 \end{array}\right]$	1행 : 원래대로 작성 3행 = 1행 $\times(-3)+$ 3행

⑤ 2행 = 2행 $\times(-1)$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -2 & -3 \\ 0 & -8 & 4 & 0 \end{array}\right] \quad \text{2행 = 2행 } \times(-1)$$

⑥ 3행 = 2행 $\times 8+$ 3행

	과정		결과	
2행 $\times 8$	$\left[\begin{array}{ccc\|c} 1 & 1 & -1 & -1 \\ 0 & 8 & -16 & -24 \\ 0 & -8 & 4 & 0 \end{array}\right]$	$\Rightarrow$	$\left[\begin{array}{ccc\|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -12 & -24 \end{array}\right]$	2행 : 원래대로 작성 3행 = 2행 $\times 8+$ 3행

⑦ 3행 = 3행 $\times\left(-\frac{1}{12}\right)$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \text{3행 = 3행 } \times\left(-\frac{1}{12}\right)$$

계수행렬이 $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$, 상수행렬이 $\begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$, 미지수행렬이 $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$일 때, 연립일차방정식은 다음과 같다.

$$\begin{cases} x+y-\ z=-1 \\ \quad\ \ y-2z=-3 \\ \qquad\quad z=\ 2 \end{cases}$$

$z=2$를 $y-2z=-3$에 대입 : $y-2\times 2=-3$ $\quad\therefore\ y=1$
$y=1,\ z=2$를 $x+y-z=-1$에 대입 : $x+1-2=-1$ $\quad\therefore\ x=0$
$\therefore$ 가우스 소거법을 이용하여 구한 연립일차방정식의 해는 $x=0,\ y=1,\ z=2$이다.

(b) 첨가행렬 : $\left[\begin{array}{ccc|c} 5 & -1 & 1 & -6 \\ 1 & 1 & 3 & 0 \\ -2 & 2 & 4 & 2 \end{array}\right]$

① 1행과 2행 자리바꿈

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 5 & -1 & 1 & -6 \\ -2 & 2 & 4 & 2 \end{array}\right]$$

② 2행 = 1행 $\times(-5)+$ 2행

과정 결과

1행 $\times(-5)$ $\left[\begin{array}{ccc|c} -5 & -5 & -15 & 0 \\ 5 & -1 & 1 & -6 \\ -2 & 2 & 4 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -6 & -14 & -6 \\ -2 & 2 & 4 & 2 \end{array}\right]$ 1행 : 원래대로 작성
2행 = 1행 $\times(-5)+$ 2행

③ 3행 = 1행 $\times 2+$ 3행

과정 결과

1행 $\times 2$ $\left[\begin{array}{ccc|c} 2 & 2 & 6 & 0 \\ 0 & -6 & -14 & -6 \\ -2 & 2 & 4 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -6 & -14 & -6 \\ 0 & 4 & 10 & 2 \end{array}\right]$ 1행 : 원래대로 작성

3행 = 1행 $\times 2+$ 3행

④ 2행 = 2행 $\times\left(-\frac{1}{6}\right)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & \frac{7}{3} & 1 \\ 0 & 4 & 10 & 2 \end{array}\right]$$ 2행 = 2행 $\times\left(-\frac{1}{6}\right)$

⑤ 3행 = 2행 $\times(-4)+$ 3행

과정 결과

2행 $\times(-4)$ $\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & -4 & -\frac{28}{3} & -4 \\ 0 & 4 & 10 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & \frac{7}{3} & 1 \\ 0 & 0 & \frac{2}{3} & -2 \end{array}\right]$ 2행 : 원래대로 작성

3행 = 2행 $\times(-4)+$ 3행

⑥ 3행 = 3행 $\times\frac{3}{2}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 0 \\ 0 & 1 & \frac{7}{3} & 1 \\ 0 & 0 & 1 & -3 \end{array}\right]$$ 3행 = 3행 $\times\frac{3}{2}$

계수행렬이 $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & \frac{7}{3} \\ 0 & 0 & 1 \end{bmatrix}$, 상수행렬이 $\begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$, 미지수행렬이 $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$일 때, 연립일차방정식은 다음과 같다.

$$\begin{cases} x+y+3z=0 \\ y+\frac{7}{3}z=1 \\ z=-3 \end{cases}$$

$z=-3$을 $y+\frac{7}{3}z=1$에 대입 : $y+\frac{7}{3}\times(-3)=1$ $\therefore\ y=8$

$y=8,\ z=-3$을 $x+y+3z=0$에 대입 : $x+8+3\times(-3)=0$ $\therefore\ x=1$

$\therefore$ 가우스 소거법을 이용하여 구한 연립일차방정식의 해는 $x=1,\ y=8,\ z=-3$이다.

(c) 첨가행렬 : $\left[\begin{array}{ccc|c} 3 & 2 & -1 & 5 \\ 1 & -3 & 4 & -9 \\ 2 & -1 & 1 & 2 \end{array}\right]$

① 1행과 2행 자리바꿈

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 3 & 2 & -1 & 5 \\ 2 & -1 & 1 & 2 \end{array}\right]$$

② 2행 = 1행 $\times(-3)$ + 2행

과정 결과

1행 $\times(-3)$ $\left[\begin{array}{ccc|c} -3 & 9 & -12 & 27 \\ 3 & 2 & -1 & 5 \\ 2 & -1 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 0 & 11 & -13 & 32 \\ 2 & -1 & 1 & 2 \end{array}\right]$ 1행 : 원래대로 작성
2행 = 1행 $\times(-3)$ + 2행

③ 3행 = 1행 $\times(-2)$ + 3행

과정 결과

1행 $\times(-2)$ $\left[\begin{array}{ccc|c} -2 & 6 & -8 & 18 \\ 0 & 11 & -13 & 32 \\ 2 & -1 & 1 & 2 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 0 & 11 & -13 & 32 \\ 0 & 5 & -7 & 20 \end{array}\right]$ 1행 : 원래대로 작성
3행 = 1행 $\times(-2)$ + 3행

④ 2행 = 2행 $\times \frac{1}{11}$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 0 & 1 & -\frac{13}{11} & \frac{32}{11} \\ 0 & 5 & -7 & 20 \end{array}\right]$$ 2행 = 2행 $\times \frac{1}{11}$

⑤ 3행 = 2행 $\times(-5)$ + 3행

과정 결과

2행 $\times(-5)$ $\left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 0 & -5 & \frac{65}{11} & -\frac{160}{11} \\ 0 & 5 & -7 & 20 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 0 & 1 & -\frac{13}{11} & \frac{32}{11} \\ 0 & 0 & -\frac{12}{11} & \frac{60}{11} \end{array}\right]$ 2행 : 원래대로 작성
3행 = 2행 $\times(-5)$ + 3행

⑥ 3행 = 3행 $\times\left(-\frac{11}{12}\right)$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -9 \\ 0 & 1 & -\frac{13}{11} & \frac{32}{11} \\ 0 & 0 & 1 & -5 \end{array}\right]$$ 3행 = 3행 $\times\left(-\frac{11}{12}\right)$

계수행렬이 $\begin{bmatrix}1 & -3 & 4\\ 0 & 1 & -\dfrac{13}{11}\\ 0 & 0 & 1\end{bmatrix}$, 상수행렬이 $\begin{bmatrix}-9\\ \dfrac{32}{11}\\ -5\end{bmatrix}$, 미지수행렬이 $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$일 때, 연립일차방정식은 다음과 같다.

$$\begin{cases} x-3y+\ 4z\ =-9 \\ \quad y-\dfrac{13}{11}z=\dfrac{32}{11} \\ \qquad\quad z\ =-5 \end{cases}$$

$z=-5$을 $y-\dfrac{13}{11}z=\dfrac{32}{11}$에 대입 : $y-\dfrac{13}{11}\times(-5)=\dfrac{32}{11}$ $\quad\therefore\ y=-3$

$y=-3$, $z=-5$을 $x-3y+4z=-9$에 대입 : $x-3\times(-3)+4\times(-5)=-9$ $\quad\therefore\ x=2$

$\therefore$ 가우스 소거법을 이용하여 구한 연립일차방정식의 해는 $x=2$, $y=-3$, $z=-5$이다.

(d) 첨가행렬 : $\left[\begin{array}{ccc|c}0 & 2 & 3 & 7\\ 2 & 6 & 8 & 10\\ 1 & 5 & 6 & 1\end{array}\right]$

① 1행과 3행 자리바꿈

$$\left[\begin{array}{ccc|c}1 & 5 & 6 & 1\\ 2 & 6 & 8 & 10\\ 0 & 2 & 3 & 7\end{array}\right]$$

② 2행 = 1행 $\times(-2)$ + 2행

과정 결과

1행 $\times(-2)$ $\left[\begin{array}{ccc|c}-2 & -10 & -12 & -2\\ 2 & 6 & 8 & 10\\ 0 & 2 & 3 & 7\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1 & 5 & 6 & 1\\ 0 & -4 & -4 & 8\\ 0 & 2 & 3 & 7\end{array}\right]$ 1행 원래대로 작성
2행 = 1행 $\times(-2)$ + 2행

③ 2행 = 2행 $\times\left(-\dfrac{1}{4}\right)$

$$\left[\begin{array}{ccc|c}1 & 5 & 6 & 1\\ 0 & 1 & 1 & -2\\ 0 & 2 & 3 & 7\end{array}\right]$$ 2행 = 2행 $\times\left(-\dfrac{1}{4}\right)$

④ 3행 = 2행 $\times(-2)$ + 3행

과정 결과

2행 $\times(-2)$ $\left[\begin{array}{ccc|c}1 & 5 & 6 & 1\\ 0 & -2 & -2 & 4\\ 0 & 2 & 3 & 7\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1 & 5 & 6 & 1\\ 0 & 1 & 1 & -2\\ 0 & 0 & 1 & 11\end{array}\right]$ 2행 원래대로 작성
3행 = 2행 $\times(-2)$ + 3행

계수행렬이 $\begin{bmatrix}1 & 5 & 6\\ 0 & 1 & 1\\ 0 & 0 & 1\end{bmatrix}$, 상수행렬이 $\begin{bmatrix}1\\ -2\\ 11\end{bmatrix}$, 미지수행렬이 $\begin{bmatrix}x\\ y\\ z\end{bmatrix}$일 때, 연립일차방정식은 다음과 같다.

$$\begin{cases} x+5y+6z=\ 1 \\ \quad y+\ z\ =-2 \\ \qquad\quad z\ =\ 11 \end{cases}$$

$z=11$을 $y+z=-2$에 대입 : $y+11=-2$ $\therefore\ y=-13$

$y=-13,\ z=11$을 $x+5y+6z=1$에 대입 : $x+5\times(-13)+6\times11=1$ $\therefore\ x=0$

$\therefore$ 가우스 소거법을 이용하여 구한 연립일차방정식의 해는 $x=0,\ y=-13,\ z=11$이다.

(2) 가우스-조단 소거법 이용

(a) "(1) 가우스 소거법 이용"의 ⑦ 이후의 단계부터 진행

⑧ 1행 = 2행 $\times(-1)+$ 1행

과정 결과

$$\text{2행}\times(-1)\quad \left[\begin{array}{ccc|c}1&1&-1&-1\\0&-1&2&3\\0&0&1&2\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&1&2\\0&1&-2&-3\\0&0&1&2\end{array}\right]$$

1행 = 2행 $\times(-1)+$ 1행
2행 : 원래대로 작성

⑨ 1행 = 3행 $\times(-1)+$ 1행

과정 결과

$$\text{3행}\times(-1)\quad \left[\begin{array}{ccc|c}1&0&1&2\\0&1&-2&-3\\0&0&-1&-2\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&0&0\\0&1&-2&-3\\0&0&1&2\end{array}\right]$$

1행 = 3행 $\times(-1)+$ 1행
3행 : 원래대로 작성

⑩ 2행 = 3행 $\times2+$ 2행

과정 결과

$$\text{3행}\times2\quad \left[\begin{array}{ccc|c}1&0&0&0\\0&1&-2&-3\\0&0&2&4\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&1\\0&0&1&2\end{array}\right]$$

2행 = 3행 $\times2+$ 2행
3행 : 원래대로 작성

$\therefore$ 가우스-조단 소거법으로 구한 해도 마찬가지로 $x=0,\ y=1,\ z=2$이다.

(b) "(1) 가우스 소거법 이용"의 ⑥ 이후의 단계부터 진행

⑦ 1행 = 2행 $\times(-1)+$ 1행

결과 과정

$$\text{2행}\times(-1)\quad \left[\begin{array}{ccc|c}1&1&3&0\\0&-1&-\frac{7}{3}&-1\\0&0&1&-3\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&\frac{2}{3}&-1\\0&1&\frac{7}{3}&1\\0&0&1&-3\end{array}\right]$$

1행 = 2행 $\times(-1)+$ 1행
2행 : 원래대로 작성

⑧ 1행 = 3행 $\times\left(-\frac{2}{3}\right)+$ 1행

과정 결과

$$\text{3행}\times\left(-\frac{2}{3}\right)\quad \left[\begin{array}{ccc|c}1&0&\frac{2}{3}&-1\\0&1&\frac{7}{3}&1\\0&0&-\frac{2}{3}&2\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c}1&0&0&1\\0&1&\frac{7}{3}&1\\0&0&1&-3\end{array}\right]$$

1행 = 3행 $\times\left(-\frac{2}{3}\right)+$ 1행

3행 : 원래대로 작성

⑨ 2행 = 3행 $\times\left(-\frac{7}{3}\right)+$ 2행

과정

3행 $\times\left(-\frac{7}{3}\right)$ $\left[\begin{array}{ccc|c}1 & 0 & 0 & 1\\ 0 & 1 & \frac{7}{3} & 1\\ 0 & 0 & -\frac{7}{3} & 7\end{array}\right]$ $\Rightarrow$

결과

$\left[\begin{array}{ccc|c}1 & 0 & 0 & 1\\ 0 & 1 & 0 & 8\\ 0 & 0 & 1 & -3\end{array}\right]$ 2행 = 3행 $\times\left(-\frac{7}{3}\right)+$ 2행

3행 : 원래대로 작성

∴ 가우스-조단 소거법으로 구한 해도 마찬가지로 $x=1,\ y=8,\ z=-3$이다.

(c) "(1) 가우스 소거법 이용"의 ⑥ 이후의 단계부터 진행

⑦ 1행 = 2행 $\times 3$ + 1행

과정

2행 $\times 3$ $\left[\begin{array}{ccc|c}1 & -3 & 4 & -9\\ 0 & 3 & -\frac{39}{11} & \frac{96}{11}\\ 0 & 0 & 1 & -5\end{array}\right]$ $\Rightarrow$

결과

$\left[\begin{array}{ccc|c}1 & 0 & \frac{5}{11} & -\frac{3}{11}\\ 0 & 1 & -\frac{13}{11} & \frac{32}{11}\\ 0 & 0 & 1 & -5\end{array}\right]$ 1행 = 2행 $\times 3$ + 1행

2행 : 원래대로 작성

⑧ 1행 = 3행 $\times\left(-\frac{5}{11}\right)$ + 1행

과정

3행 $\times\left(-\frac{5}{11}\right)$ $\left[\begin{array}{ccc|c}1 & 0 & \frac{5}{11} & -\frac{3}{11}\\ 0 & 1 & -\frac{13}{11} & \frac{32}{11}\\ 0 & 0 & -\frac{5}{11} & \frac{25}{11}\end{array}\right]$ $\Rightarrow$

결과

$\left[\begin{array}{ccc|c}1 & 0 & 0 & \frac{22}{11}\\ 0 & 1 & -\frac{13}{11} & \frac{32}{11}\\ 0 & 0 & 1 & -5\end{array}\right]$ 1행 = 3행 $\times\left(-\frac{5}{11}\right)$ + 1행

3행 : 원래대로 작성

⑨ 1행 = 3행 $\times\frac{13}{11}$ + 1행

과정

3행 $\times\frac{13}{11}$ $\left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\ 0 & 1 & -\frac{13}{11} & \frac{32}{11}\\ 0 & 0 & \frac{13}{11} & -\frac{65}{11}\end{array}\right]$ $\Rightarrow$

결과

$\left[\begin{array}{ccc|c}1 & 0 & 0 & 2\\ 0 & 1 & 0 & -3\\ 0 & 0 & 1 & -5\end{array}\right]$ 1행 = 3행 $\times\frac{13}{11}$ + 1행

3행 : 원래대로 작성

∴ 가우스-조단 소거법을 이용하여 구한 해도 $x=2,\ y=-3,\ z=-5$이다.

(d) "(1) 가우스 소거법 이용"의 ④ 이후의 단계부터 진행

⑤ 1행 = 2행 $\times(-5)$ + 1행

과정

2행 $\times(-5)$ $\left[\begin{array}{ccc|c}1 & 5 & 6 & 1\\ 0 & -5 & -5 & 10\\ 0 & 0 & 1 & 11\end{array}\right]$ $\Rightarrow$

결과

$\left[\begin{array}{ccc|c}1 & 0 & 1 & 11\\ 0 & 1 & 1 & -2\\ 0 & 0 & 1 & 11\end{array}\right]$ 1행 = 2행 $\times(-5)$ + 1행

2행 원래대로 작성

⑥ 1행 = 3행 $\times(-1)$ + 1행

$$
\begin{array}{ccccc}
 & \text{과정} & & \text{결과} & \\
3\text{행}\ \times(-1) & \left[\begin{array}{ccc|c} 1 & 0 & 1 & 11 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -1 & -11 \end{array}\right] & \Rightarrow & \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & 11 \end{array}\right] & \begin{array}{l} 1\text{행} = 3\text{행} \times(-1) + 1\text{행} \\ \\ 3\text{행 원래대로 작성} \end{array}
\end{array}
$$

⑦ 2행 = 3행 $\times(-1)$ + 2행

$$
\begin{array}{ccccc}
 & \text{과정} & & \text{결과} & \\
3\text{행}\ \times(-1) & \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -1 & -11 \end{array}\right] & \Rightarrow & \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -13 \\ 0 & 0 & 1 & 11 \end{array}\right] & \begin{array}{l} 2\text{행} = 3\text{행} \times(-1) + 2\text{행} \\ 3\text{행 원래대로 작성} \end{array}
\end{array}
$$

∴ 가우스-조단 소거법을 이용하여 구한 해도 $x = 0$, $y = -13$, $z = 11$이다.

16.

(a) $x = -17$, $y = 1$, $z = -8$

(b) $x = -\dfrac{13}{2}$, $y = \dfrac{7}{2}$, $z = 7$

(c) $x = -3$, $y = 8$, $z = 1$

(d) $x = 6$, $y = -5$, $z = 2$

17.

(a) $w = -3$, $x = 1$, $y = 0$, $z = 5$

(b) $w = -1$, $x = 2$, $y = 5$, $z = 0$

(c) $w = -7$, $x = -1$, $y = 4$, $z = 2$

(d) $w = 5$, $x = -6$, $y = 2$, $z = 4$