6장 연습문제 정답

01.

(a)

$R: A \times A$

$R: A \times B$

$R: B \times A$

$R: B \times B$

(b)

$R: A \times A \times A$

$R: A \times A \times B$

$R: A \times A \times C$

$R: A \times B \times A$

$R: A \times B \times B$

$R: A \times B \times C$

$R: A \times C \times A$

$R: A \times C \times B$

$R: A \times C \times C$

$R: B \times A \times A$

$R: B \times A \times B$

$R: B \times A \times C$

$R: B \times B \times A$

$R: B \times B \times B$

$R: B \times B \times C$

$R: B \times C \times A$

$R: B \times C \times B$

$R: B \times C \times C$

$R: C \times A \times A$

$R: C \times A \times B$

$R: C \times A \times C$

$R: C \times B \times A$

$R: C \times B \times B$

$R: C \times B \times C$

$R: C \times C \times A$

$R: C \times C \times B$

$R: C \times C \times C$

02.

(a)

${}_1R_{1_a}$	$\because\ (1,a)\in R_1$	${}_1\cancel{R}_{1_b}$	$\because\ (1,b)\notin R_1$	${}_1R_{1_c}$	$\because\ (1,c)\in R_1$
${}_2\cancel{R}_{1a}$	$\because\ (2,a)\notin R_1$	${}_2R_{1_b}$	$\because\ (2,b)\in R_1$	${}_2\cancel{R}_{1c}$	$\because\ (2,c)\notin R_1$
${}_3\cancel{R}_{1a}$	$\because\ (3,a)\notin R_1$	${}_3\cancel{R}_{1b}$	$\because\ (3,b)\notin R_1$	${}_3R_{1_c}$	$\because\ (3,c)\in R_1$
${}_4R_{1_a}$	$\because\ (4,a)\in R_1$	${}_4R_{1_b}$	$\because\ (4,b)\in R_1$	${}_4\cancel{R}_{1c}$	$\because\ (4,c)\notin R_1$
${}_1R_{2_a}$	$\because\ (1,a)\in R_2$	${}_1\cancel{R}_{2_b}$	$\because\ (1,b)\notin R_2$	${}_1\cancel{R}_{2_c}$	$\because\ (1,c)\notin R_2$
${}_2R_{2_a}$	$\because\ (2,a)\in R_2$	${}_2R_{2_b}$	$\because\ (2,b)\in R_2$	${}_2\cancel{R}_{2c}$	$\because\ (2,c)\notin R_2$
${}_3\cancel{R}_{2a}$	$\because\ (3,a)\notin R_2$	${}_3R_{2_b}$	$\because\ (3,b)\in R_2$	${}_3\cancel{R}_{2c}$	$\because\ (3,c)\notin R_2$
${}_4\cancel{R}_{2a}$	$\because\ (4,a)\notin R_2$	${}_4\cancel{R}_{2b}$	$\because\ (4,b)\notin R_2$	${}_4\cancel{R}_{1c}$	$\because\ (4,c)\notin R_1$
${}_1\cancel{R}_{3_a}$	$\because\ (1,a)\notin R_3$	${}_1\cancel{R}_{3_b}$	$\because\ (1,b)\notin R_3$	${}_1\cancel{R}_{3_c}$	$\because\ (1,c)\notin R_3$
${}_2\cancel{R}_{3_a}$	$\because\ (2,a)\notin R_3$	${}_2\cancel{R}_{3_b}$	$\because\ (2,b)\notin R_3$	${}_2R_{3_c}$	$\because\ (2,c)\in R_3$
${}_3\cancel{R}_{3_a}$	$\because\ (3,a)\notin R_3$	${}_3\cancel{R}_{3_b}$	$\because\ (3,b)\notin R_3$	${}_3\cancel{R}_{3_c}$	$\because\ (3,c)\notin R_3$
${}_4\cancel{R}_{3_a}$	$\because\ (4,a)\notin R_3$	${}_4\cancel{R}_{3_b}$	$\because\ (4,b)\notin R_3$	${}_4R_{3_c}$	$\because\ (4,c)\in R_3$
${}_1\cancel{R}_{4_a}$	$\because\ (1,a)\notin R_4$	${}_1R_{4_b}$	$\because\ (1,b)\in R_4$	${}_1R_{4_c}$	$\because\ (1,c)\in R_4$
${}_2R_{4_a}$	$\because\ (2,a)\in R_4$	${}_2\cancel{R}_{4_b}$	$\because\ (2,b)\notin R_4$	${}_2R_{4_c}$	$\because\ (2,c)\in R_4$
${}_3R_{4_a}$	$\because\ (3,a)\in R_4$	${}_3R_{4_b}$	$\because\ (3,b)\in R_4$	${}_3\cancel{R}_{4_c}$	$\because\ (3,c)\notin R_4$
${}_4\cancel{R}_{4_a}$	$\because\ (4,a)\notin R_4$	${}_4\cancel{R}_{4_b}$	$\because\ (4,b)\notin R_4$	${}_4\cancel{R}_{4_c}$	$\because\ (4,c)\notin R_4$
${}_1R_{5_a}$	$\because\ (1,a)\in R_5$	${}_1R_{5_b}$	$\because\ (1,b)\in R_5$	${}_1R_{5_c}$	$\because\ (1,c)\in R_5$
${}_2\cancel{R}_{5_a}$	$\because\ (2,a)\notin R_5$	${}_2R_{5_b}$	$\because\ (2,b)\in R_5$	${}_2\cancel{R}_{5_c}$	$\because\ (2,c)\notin R_5$
${}_3\cancel{R}_{5_a}$	$\because\ (3,a)\notin R_5$	${}_3R_{5_b}$	$\because\ (3,b)\in R_5$	${}_3\cancel{R}_{5_c}$	$\because\ (3,c)\notin R_5$
${}_4R_{5_a}$	$\because\ (4,a)\in R_5$	${}_4R_{5_b}$	$\because\ (4,b)\in R_5$	${}_4R_{5_c}$	$\because\ (4,c)\in R_5$

(b)

$dom(R_1) = dom(R_2) = dom(R_3) = dom(R_4) = dom(R_5) = A$

$codom(R_1) = codom(R_2) = codom(R_3) = codom(R_4) = codom(R_5) = B$

$ran(R_1) = \{a, b, c\}$ $\quad ran(R_2) = \{a, b\}$ $\quad ran(R_3) = \{c\}$ $\quad ran(R_4) = \{a, b, c\}$ $\quad ran(R_5) = \{a, b, c\}$

(c)

$R_1^{-1} = \{(a,1), (a,4), (b,2), (b,4), (c,1), (c,3)\}$

$R_2^{-1} = \{(a,1), (a,2), (b,2), (b,3)\}$

$R_3^{-1} = \{(c,2), (c,4)\}$

$R_4^{-1} = \{(a,2), (a,3), (b,1), (b,3), (c,1), (c,2)\}$

$R_5^{-1} = \{(a,1), (a,4), (b,1), (b,2), (b,3), (b,4), (c,1), (c,4)\}$

(d)

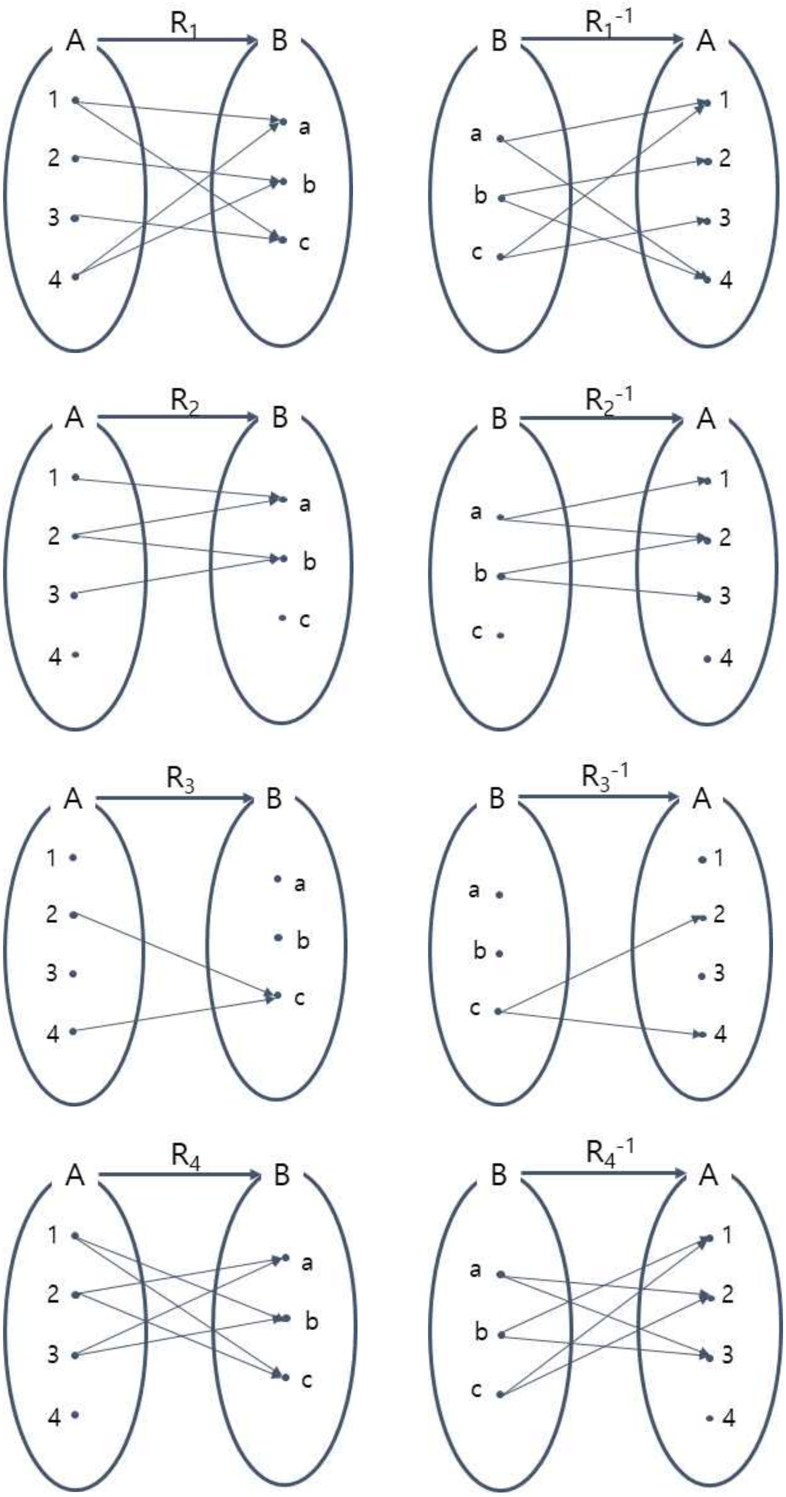
$dom(R_1^{-1}) = dom(R_2^{-1}) = dom(R_3^{-1}) = dom(R_4^{-1}) = dom(R_5^{-1}) = B$

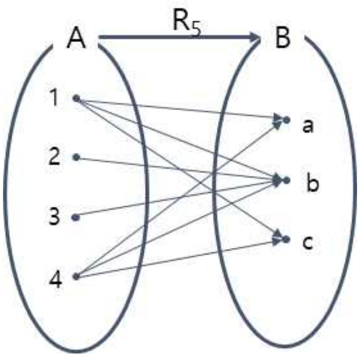
$codom(R_1^{-1}) = codom(R_2^{-1}) = codom(R_3^{-1}) = codom(R_4^{-1}) = codom(R_5^{-1}) = A$

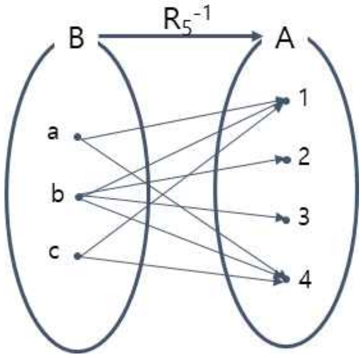
$ran(R_1^{-1}) = \{1, 2, 3, 4\}$ $\quad ran(R_2^{-1}) = \{1, 2, 3\}$ $\quad ran(R_3^{-1}) = \{2, 4\}$

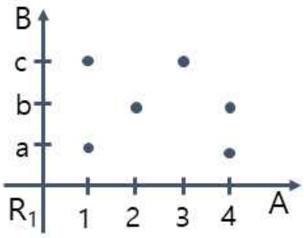
$ran(R_4^{-1}) = \{1, 2, 3\}$ $\quad ran(R_5^{-1}) = \{1, 2, 3, 4\}$

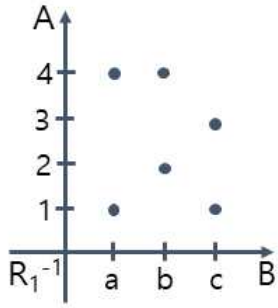
(e)

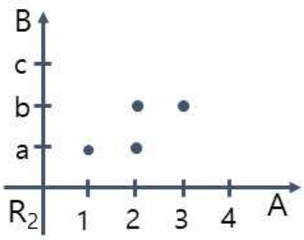


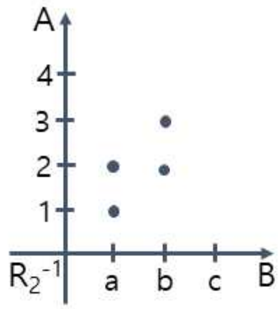

R_5
A
B
1
2
3
4
a
b
c

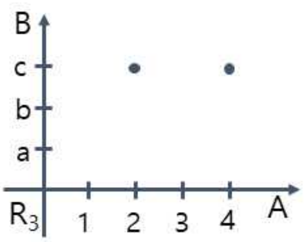

R_5^{-1}
B
A
a
b
c
1
2
3
4

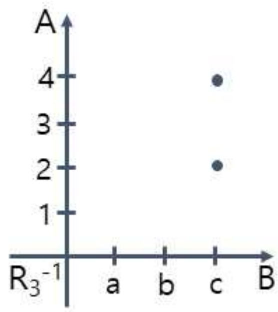

B
c
b
a
R_1
1
2
3
4
A

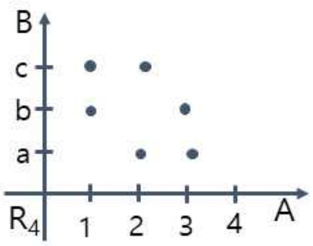

A
4
3
2
1
R_1^{-1}
a
b
c
B

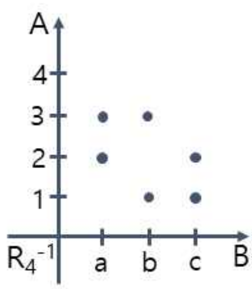

B
c
b
a
R_2
1
2
3
4
A

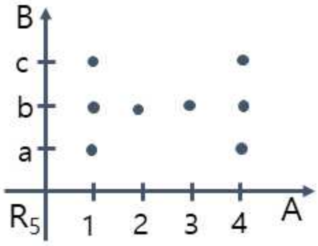

A
4
3
2
1
R_2^{-1}
a
b
c
B

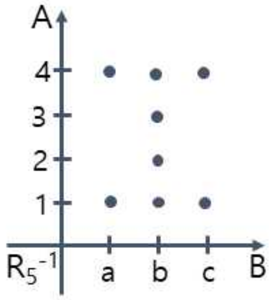

B
c
b
a
R_3
1
2
3
4
A


A
4
3
2
1
R_3^{-1}
a
b
c
B


B
c
b
a
R_4
1
2
3
4
A


A
4
3
2
1
R_4^{-1}
a
b
c
B





$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array} \qquad M_{R_1^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$M_{R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad M_{R_2^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R_3} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad M_{R_3^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$$M_{R_4} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array} \qquad M_{R_4^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R_5} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \qquad M_{R_5^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

03.

(a)

$a\,R_1\,w$ $\quad$ $b\,R_1\,w$ $\quad$ $c\,\cancel{R_1}\,w$ $\quad$ $d\,\cancel{R_1}\,w$
$a\,\cancel{R_1}\,x$ $\quad$ $b\,R_1\,x$ $\quad$ $c\,R_1\,x$ $\quad$ $d\,R_1\,x$
$a\,R_1\,y$ $\quad$ $b\,\cancel{R_1}\,y$ $\quad$ $c\,\cancel{R_1}\,y$ $\quad$ $d\,R_1\,y$
$a\,\cancel{R_1}\,z$ $\quad$ $b\,R_1\,z$ $\quad$ $c\,R_1\,z$ $\quad$ $d\,\cancel{R_1}\,z$

$a\,\cancel{R_2}\,w$ $\quad$ $b\,R_2\,w$ $\quad$ $c\,R_2\,w$ $\quad$ $d\,\cancel{R_2}\,w$
$a\,R_2\,x$ $\quad$ $b\,\cancel{R_2}\,x$ $\quad$ $c\,R_2\,x$ $\quad$ $d\,R_2\,x$
$a\,R_2\,y$ $\quad$ $b\,\cancel{R_2}\,y$ $\quad$ $c\,\cancel{R_2}\,y$ $\quad$ $d\,R_2\,y$
$a\,R_2\,z$ $\quad$ $b\,R_2\,z$ $\quad$ $c\,R_2\,z$ $\quad$ $d\,\cancel{R_2}\,z$

$a\,\cancel{R_3}\,w$ $\quad$ $b\,\cancel{R_3}\,w$ $\quad$ $c\,R_3\,w$ $\quad$ $d\,\cancel{R_3}\,w$
$a\,\cancel{R_3}\,x$ $\quad$ $b\,R_3\,x$ $\quad$ $c\,\cancel{R_3}\,x$ $\quad$ $d\,\cancel{R_3}\,x$
$a\,\cancel{R_3}\,y$ $\quad$ $b\,\cancel{R_3}\,y$ $\quad$ $c\,\cancel{R_3}\,y$ $\quad$ $d\,R_3\,y$
$a\,R_3\,z$ $\quad$ $b\,\cancel{R_3}\,z$ $\quad$ $c\,\cancel{R_3}\,z$ $\quad$ $d\,\cancel{R_3}\,z$

$a\,R_4\,w$ $\quad$ $b\,R_4\,w$ $\quad$ $c\,\cancel{R_4}\,w$ $\quad$ $d\,R_4\,w$
$a\,\cancel{R_4}\,x$ $\quad$ $b\,R_4\,x$ $\quad$ $c\,\cancel{R_4}\,x$ $\quad$ $d\,\cancel{R_4}\,x$
$a\,\cancel{R_4}\,y$ $\quad$ $b\,R_4\,y$ $\quad$ $c\,R_4\,y$ $\quad$ $d\,R_4\,y$
$a\,R_4\,z$ $\quad$ $b\,R_4\,z$ $\quad$ $c\,R_4\,z$ $\quad$ $d\,\cancel{R_4}\,z$

$a \cancel{R_5} w$ $\quad b \cancel{R_5} w$ $\quad c R_5 w$ $\quad d \cancel{R_5} w$

$a \cancel{R_5} x$ $\quad b R_5 x$ $\quad c \cancel{R_5} x$ $\quad d R_5 x$

$a R_5 y$ $\quad b \cancel{R_5} y$ $\quad c R_5 y$ $\quad d R_5 y$

$a R_5 z$ $\quad b R_5 z$ $\quad c \cancel{R_5} z$ $\quad d \cancel{R_5} z$

(b)

$dom(R_1) = dom(R_2) = dom(R_3) = dom(R_4) = dom(R_5) = A$

$codom(R_1) = codom(R_2) = codom(R_3) = codom(R_4) = codom(R_5) = B$

$ran(R_1) = ran(R_2) = ran(R_3) = ran(R_4) = ran(R_5) = B$

(c)

$R_1^{-1} = \{(w,a), (w,b), (x,b), (x,c), (x,d), (y,a), (y,d), (z,b), (z,c)\}$

$R_2^{-1} = \{(w,b), (w,c), (x,a), (x,c), (x,d), (y,a), (y,d), (z,a), (z,b), (z,c)\}$

$R_3^{-1} = \{(w,c), (x,b), (y,d), (z,a)\}$

$R_4^{-1} = \{(w,a), (w,b), (w,d), (x,b), (y,c), (y,d), (z,a), (z,b), (z,c)\}$

$R_5^{-1} = \{(w,c), (x,b), (x,d), (y,a), (y,c), (y,d), (z,a), (z,b)\}$

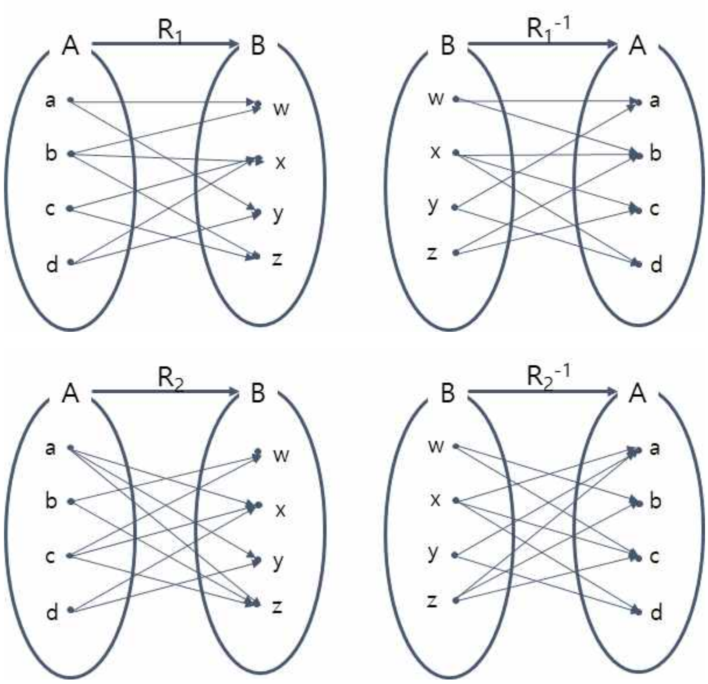
(d)

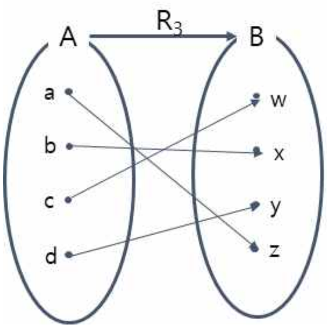
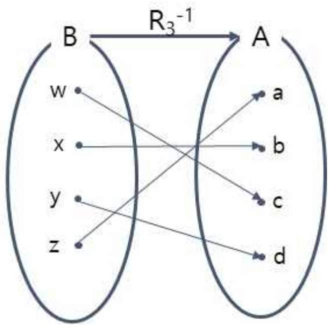
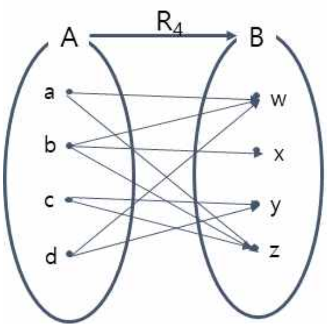
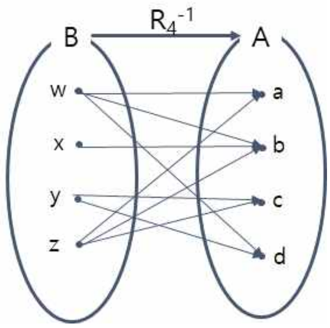
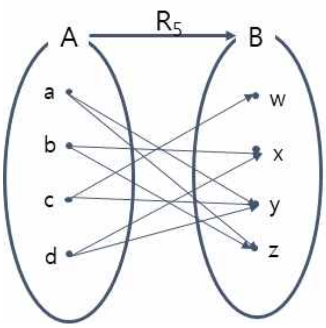
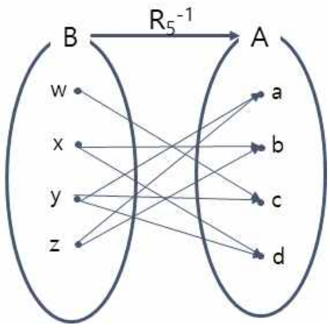
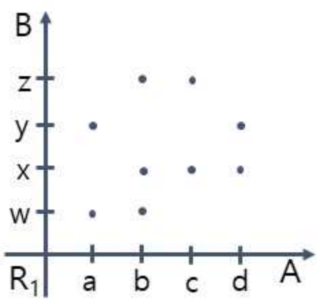
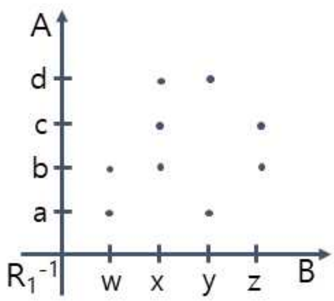
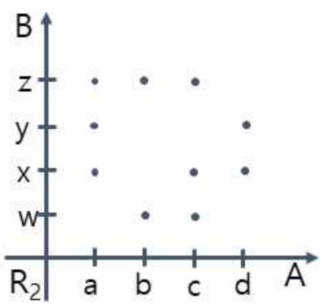
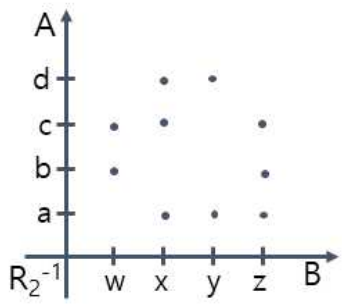
$dom(R_1^{-1}) = dom(R_2^{-1}) = dom(R_3^{-1}) = dom(R_4^{-1}) = dom(R_5^{-1}) = B$

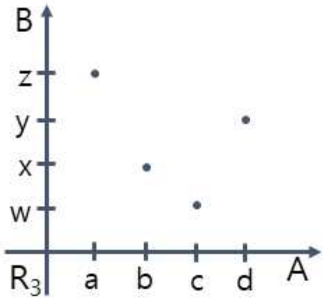
$codom(R_1^{-1}) = codom(R_2^{-1}) = codom(R_3^{-1}) = codom(R_4^{-1}) = codom(R_5^{-1}) = A$

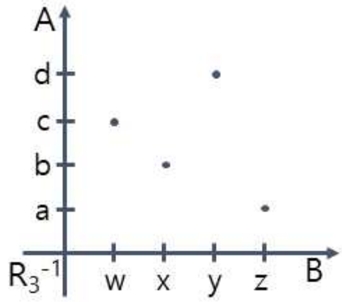
$ran(R_1^{-1}) = ran(R_2^{-1}) = ran(R_3^{-1}) = ran(R_4^{-1}) = ran(R_5^{-1}) = A$

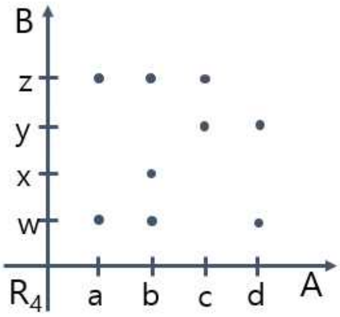
(e)

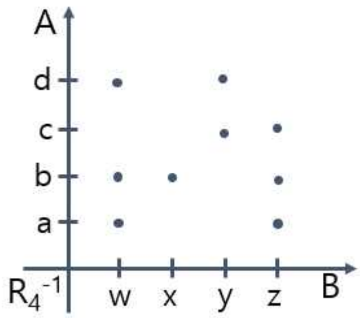


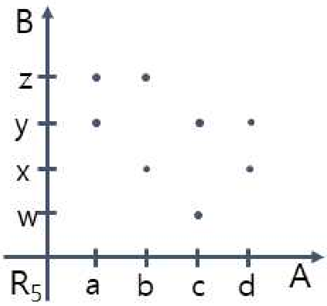

A
R_3
B
a
b
c
d
w
x
y
z

B
R_3^{-1}
A
w
x
y
z
a
b
c
d

A
R_4
B
a
b
c
d
w
x
y
z

B
R_4^{-1}
A
w
x
y
z
a
b
c
d

A
R_5
B
a
b
c
d
w
x
y
z

B
R_5^{-1}
A
w
x
y
z
a
b
c
d

B
z
y
x
w
R_1
a
b
c
d
A

A
d
c
b
a
R_1^{-1}
w
x
y
z
B

B
z
y
x
w
R_2
a
b
c
d
A

A
d
c
b
a
R_2^{-1}
w
x
y
z
B

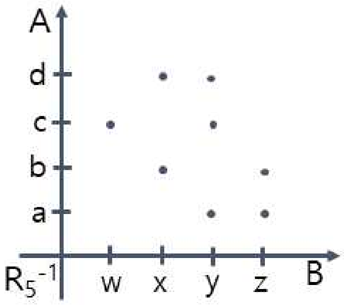












$$M_{R_1} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad M_{R_1^{-1}} = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$M_{R_2} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad M_{R_1^{-1}} = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

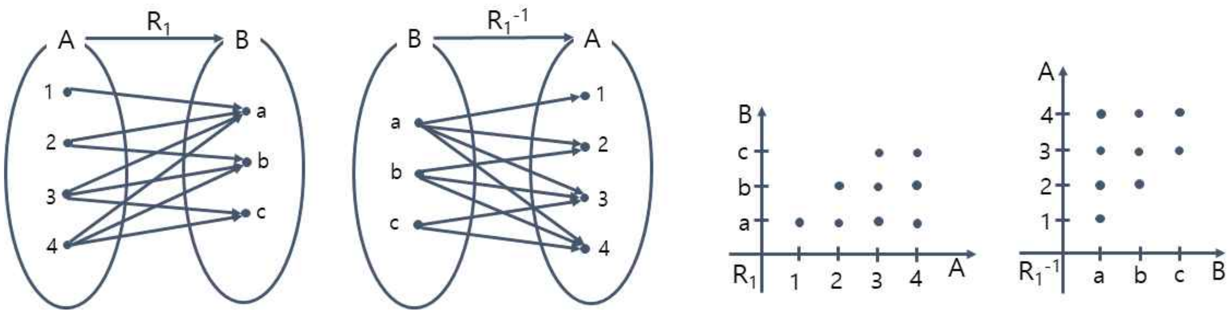
$$M_{R_3} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad M_{R_3^{-1}} = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$M_{R_4} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad M_{R_1^{-1}} = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$$M_{R_5} = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\!\!\begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{array} \qquad M_{R_5^{-1}} = \begin{array}{c} \\ w \\ x \\ y \\ z \end{array}\!\!\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

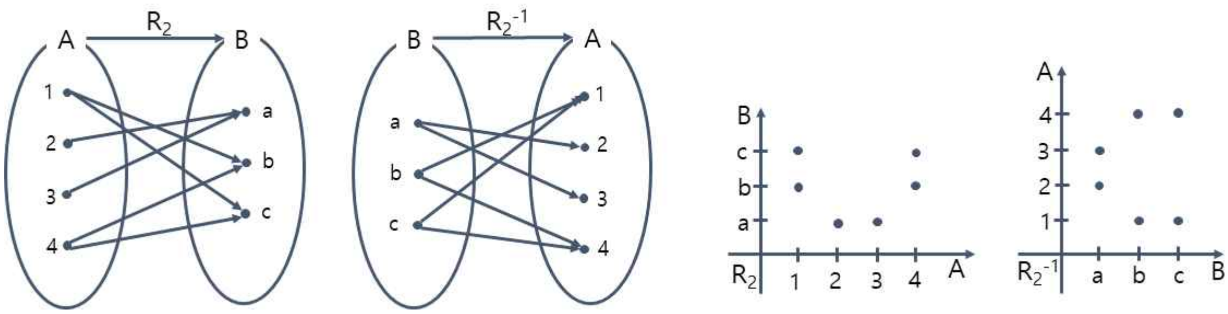
04.

(a)



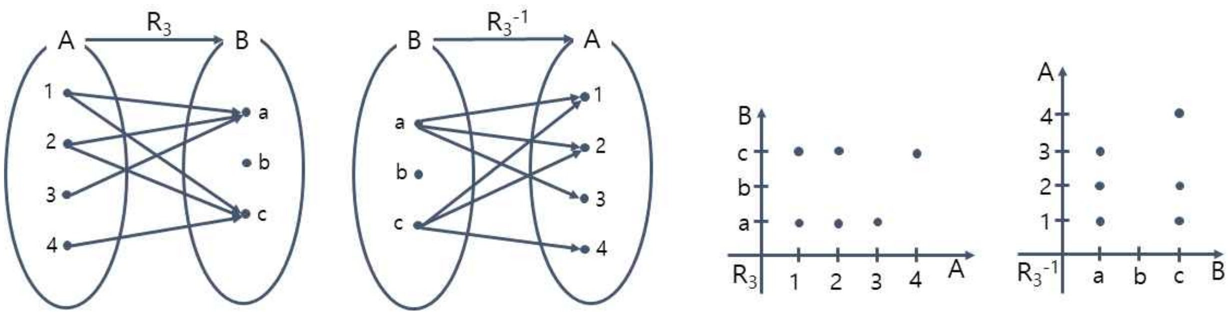
$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \end{array} \qquad M_{R_1^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

(b)



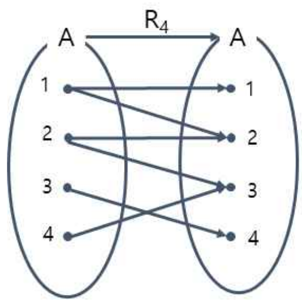
$$M_{R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \qquad M_{R_2^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

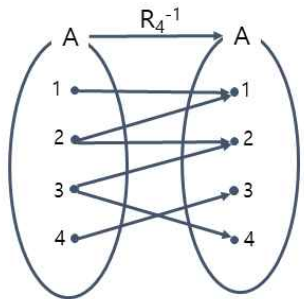
(c)

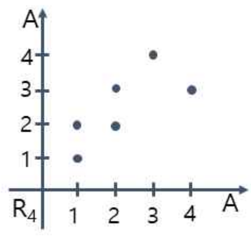


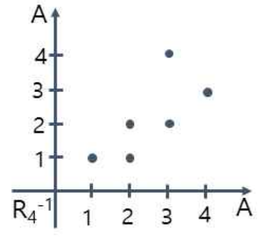
$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \qquad M_{R_2^{-1}} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

(d)

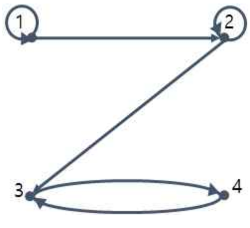




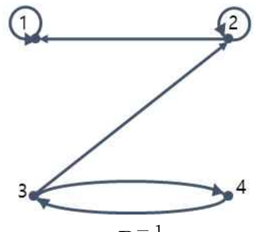




$$M_{R_4} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \quad M_{R_4^{-1}} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

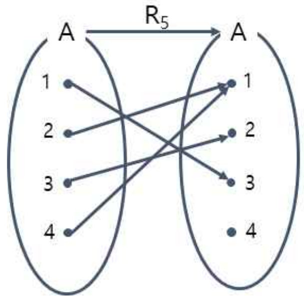


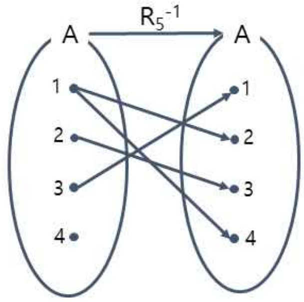
R_4

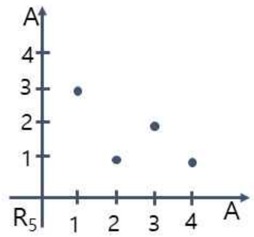


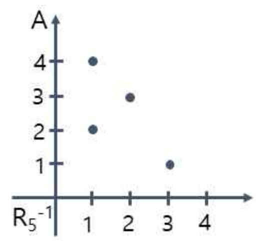
R_4^{-1}

(e)

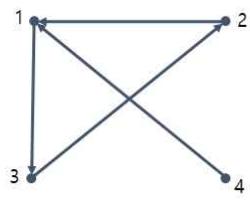




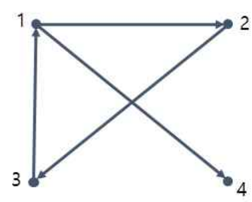




$$M_{R_5} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad M_{R_5^{-1}} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\!\!\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

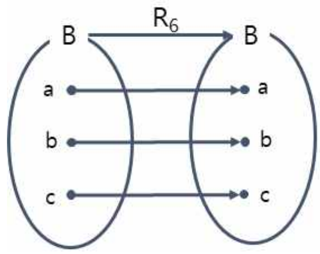


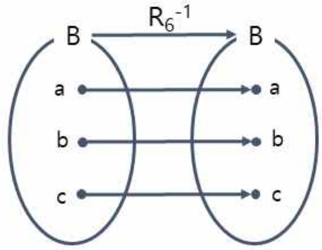
R_5

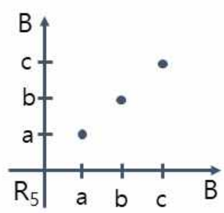


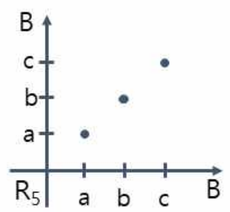
R_5^{-1}

(f)

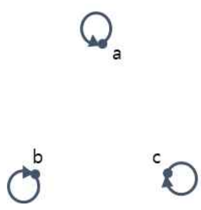




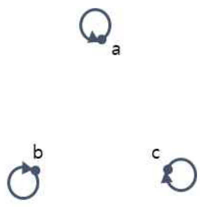




$$M_{R_6} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \quad M_{R_6^{-1}} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$



R_6



R_6^{-1}

05.

(a)

$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$(1,1) \in R_1$, $(2,2) \in R_1$, $(3,3) \in R_1$, $(4,4) \in R_1$

$\therefore$ 반사관계이고 비반사관계는 아니다.

$(1,1) \in R_1$일 때 $1=1$이고 $(1,1) \in R_1$

$(2,2) \in R_1$일 때 $2=2$이고 $(2,2) \in R_1$

$(3,3) \in R_1$일 때 $3=3$이고 $(3,3) \in R_1$

$(4,4) \in R_1$일 때 $4=4$이고 $(4,4) \in R_1$

$(2,1) \in R_1$일 때, $2 \neq 1$이고 $(1,2) \notin R_1$

$(3,1) \in R_1$일 때, $3 \neq 1$이고 $(1,3) \notin R_1$

$(4,1) \in R_1$일 때, $4 \neq 1$이고 $(1,4) \notin R_1$

$\therefore$ 대칭관계는 아니고, 반대칭관계이다.

$(1,1) \in R_1$이고 $(1,1) \in R_1$일 때 $(1,1) \in R_1$

$(2,2) \in R_1$이고 $(2,2) \in R_1$일 때 $(2,2) \in R_1$

$(3,1) \in R_1$이고 $(1,1) \in R_1$일 때 $(3,1) \in R_1$

$(3,3) \in R_1$이고 $(3,3) \in R_1$일 때 $(3,3) \in R_1$

$(4,4) \in R_1$이고 $(4,1) \in R_1$일 때 $(4,1) \in R_1$

$(2,1) \in R_1$이고 $(1,1) \in R_1$일 때 $(2,1) \in R_1$

$(2,2) \in R_1$이고 $(2,1) \in R_1$일 때 $(2,1) \in R_1$

$(3,3) \in R_1$이고 $(3,1) \in R_1$일 때 $(3,1) \in R_1$

$(4,1) \in R_1$이고 $(1,1) \in R_1$일 때 $(4,1) \in R_1$

$(4,4) \in R_1$이고 $(4,4) \in R_1$일 때 $(4,4) \in R_1$

$\therefore$ 추이관계이다.

(b)

$$M_{R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$(1,1) \notin R_2$, $(2,2) \in R_2$, $(3,3) \notin R_2$, $(4,4) \notin R_2$

$\therefore$ 반사관계는 아니고 비반사관계도 아니다.

$(2,2) \in R_2$일 때 $2=2$이고 $(2,2) \in R_2$

$(3,2) \in R_2$일 때 $3 \neq 2$이고 $(2,3) \in R_2$

$(2,3) \in R_2$일 때 $2 \neq 3$이고 $(3,2) \in R_2$

$\therefore$ 대칭관계이고, 반대칭관계는 아니다.

$(2,2) \in R_2$이고 $(2,2) \in R_2$일 때 $(2,2) \in R_2$ $(2,2) \in R_2$이고 $(2,3) \in R_2$일 때 $(2,3) \in R_2$

$(2,3) \in R_2$이고 $(3,2) \in R_2$일 때 $(2,2) \in R_2$ $(3,2) \in R_2$이고 $(2,3) \in R_2$일 때 $(3,3) \notin R_2$

∴ 추이관계 아니다.

(c)

$$M_{R_3} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$(1,1) \in R_3$, $(2,2) \in R_3$, $(3,3) \in R_3$, $(4,4) \in R_3$

∴ 반사관계이고 비반사관계는 아니다.

$(1,1) \in R_3$일 때 $1 = 1$이고 $(1,1) \in R_3$ $(2,2) \in R_3$일 때 $2 = 2$이고 $(2,2) \in R_3$

$(3,3) \in R_3$일 때 $3 = 3$이고 $(3,3) \in R_3$ $(4,4) \in R_4$일 때 $4 = 4$이고 $(4,4) \in R_4$

∴ 대칭관계이고 반대칭관계이다.

$(1,1) \in R_3$이고 $(1,1) \in R_3$일 때 $(1,1) \in R_3$ $(2,2) \in R_3$이고 $(2,2) \in R_3$일 때 $(2,2) \in R_3$

$(3,3) \in R_3$이고 $(3,3) \in R_3$일 때 $(3,3) \in R_3$ $(4,4) \in R_4$이고 $(4,4) \in R_4$일 때 $(4,4) \in R_4$

∴ 추이관계이다.

(d)

$$M_{R_4} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

$(1,1) \in R_4$, $(2,2) \in R_4$, $(3,3) \in R_4$, $(4,4) \in R_4$

∴ 반사관계이고 비반사관계는 아니다.

$(1,1) \in R_4$일 때 $1 = 1$이고 $(1,1) \in R_4$ $(1,2) \in R_4$일 때 $1 \neq 2$이고 $(2,1) \in R_4$

$(2,1) \in R_4$일 때 $2 \neq 1$이고 $(1,2) \in R_4$ $(2,2) \in R_4$일 때 $2 = 2$이고 $(2,2) \in R_4$

$(2,3) \in R_4$일 때 $2 \neq 3$이고 $(3,2) \in R_4$ $(3,2) \in R_4$일 때 $3 \neq 2$이고 $(2,3) \in R_4$

$(3,3) \in R_4$일 때 $3 = 3$이고 $(3,3) \in R_4$ $(3,4) \in R_4$일 때 $3 \neq 3$이고 $(4,3) \in R_4$

$(4,3) \in R_4$일 때 $4 \neq 3$이고 $(3,4) \in R_4$ $(4,4) \in R_4$일 때 $4 = 4$이고 $(4,4) \in R_4$

∴ 대칭관계이고 반대칭관계는 아니다.

$(1,1) \in R_4$이고 $(1,1) \in R_4$일 때 $(1,1) \in R_4$ $(1,2) \in R_4$이고 $(2,1) \in R_4$일 때 $(1,1) \in R_4$

$(1,2) \in R_4$이고 $(2,2) \in R_4$일 때 $(1,2) \in R_4$ $(1,2) \in R_4$이고 $(2,3) \in R_4$일 때 $(1,3) \notin R_4$

∴ 추이관계가 아니다.

(e)

$$M_{R_5}=\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

$(1,1)\notin R_5$, $(2,2)\notin R_5$, $(3,3)\notin R_5$, $(4,4)\notin R_5$

$\therefore$ 반사관계는 아니고 비반사관계이다.

$(1,2)\in R_5$일 때 $2\neq 1$이고 $(2,1)\notin R_5$

$(2,4)\in R_5$일 때 $2\neq 4$이고 $(4,2)\notin R_5$

$(3,1)\in R_5$일 때 $3\neq 1$이고 $(1,3)\notin R_5$

$(4,1)\in R_5$일 때 $4\neq 1$이고 $(1,4)\notin R_5$

$\therefore$ 대칭관계는 아니고 반대칭관계이다.

$(1,2)\in R_5$이고 $(2,4)\in R_5$일 때 $(1,4)\notin R_5$

$\therefore$ 추이관계가 아니다.

06.

(a) 반사관계도 아니고 비반사관계도 아니다.
대칭관계는 아니고 반대칭관계이다.
추이관계이다.

(b) 반사관계도 아니고 비반사관계도 아니다.
대칭관계이고, 반대칭관계이다.
추이관계이다.

(c) 반사관계이고 비반사관계는 아니다.
대칭관계는 아니고 반대칭관계이다.
추이관계이다.

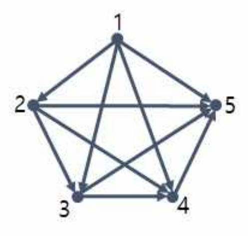
(d) 반사관계도 아니고 비반사관계도 아니다.
대칭관계이고 반대칭관계는 아니다.
추이관계가 아니다.

(e) 반사관계는 아니고 비반사관계이다.
대칭관계도 아니고 반대칭관계도 아니다.
추이관계가 아니다.

07.

(a)

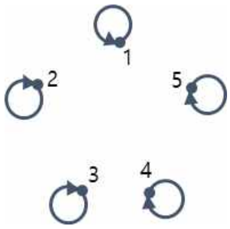
$$M_{R_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



∴ 반사관계×. 비반사관계○, 대칭관계×, 반대칭관계○, 추이관계○

(b)

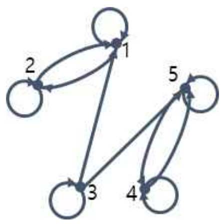
$$M_{R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



∴ 반사관계○, 비반사관계×, 대칭관계○, 반대칭관계○, 추이관계○

(c)

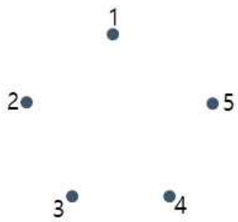
$$M_{R_3} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$



∴ 반사관계○, 비반사관계×, 대칭관계×, 반대칭관계×, 추이관계×

(d) $R_4 = \varnothing$

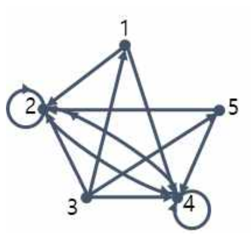
$$M_{R_4} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



∴ 반사관계×, 비반사관계○, 대칭관계○, 반대칭관계○, 추이관계○

(e)

$$M_{R_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\!\!\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



∴ 반사관계×, 비반사관계×, 대칭관계×, 반대칭관계×, 추이관계○

08.

(a)

① $R \circ R$: 앞의 R의 정의역(A)과 뒤의 R의 공역(B)이 다르므로 합성 불가

② $R \circ S$: 앞의 R의 정의역(A)과 뒤의 S의 공역(C)이 다르므로 합성 불가

④ $R \circ U$: 앞의 R의 정의역(A)과 뒤의 U의 공역(C)이 다르므로 합성 불가

⑥ $R \circ W$: 앞의 R의 정의역(A)과 뒤의 W의 공역(B)이 다르므로 합성 불가

⑦ $S \circ U$: 앞의 S의 정의역(A)과 뒤의 U의 공역(C)이 다르므로 합성 불가

⑨ $S \circ W$: 앞의 S의 정의역(A)과 뒤의 W의 공역(B)이 다르므로 합성 불가

⑪ $T \circ T$: 앞의 T의 정의역(B)과 뒤의 T의 공역(A)이 다르므로 합성 불가

⑫ $T \circ V$: 앞의 T의 정의역(B)과 뒤의 V의 공역(A)이 다르므로 합성 불가

(b)

③ $R \circ T$: $B \to B = \{(w,x),(w,y),(w,z),(x,y),(x,z),(y,x),(y,y),(z,w),(z,x),(z,y)\}$

$dom(R \circ T) = B$ $\quad$ $codom(R \circ T) = B$ $\quad$ $ran(R \circ T) = \{w,x,y,z\} = B$

⑤ $R \circ V$: $C \to B = \{(o,x),(o,y),(o,z),(p,x),(p,y),(p,z),(q,w),(q,x),(q,y),(r,w),(r,x),(r,y)\}$

$dom(R \circ V) = C$ $\quad$ $codom(R \circ V) = B$ $\quad$ $ran(R \circ T) = \{w,x,y,z\} = B$

⑧ $S \circ V : C \to C : \{(o,q),(o,r),(p,q),(p,r),(q,o),(q,p),(q,q),(q,r),(r,o),(r,p),(r,q),(r,r)\}$

$dom(S \circ V) = C$ $\quad$ $codom(S \circ V) = C$ $\quad$ $ran(S \circ V) = \{o,p,q,r\} = C$

⑩ $T \circ R$: $A \to A = \{(a,b),(a,c),(a,d),(b,c),(b,d),(c,b),(c,c),(d,a),(d,b),(d,c)\}$

$dom(T \circ R) = A$ $\quad$ $codom(T \circ R) = A$ $\quad$ $ran(T \circ R) = \{a,b,c,d\} = A$

09.

(a)

① $(W \circ S) \circ R$: 합성 불가

② $T \circ (U \circ W)$: 합성 불가

⑧ $U \circ (W \circ T)$: 합성 불가

⑨ $(U \circ W) \circ (R \circ V)$: 합성 불가

⑩ $(W \circ U) \circ (R \circ U)$: 합성 불가

(b)

③ $W \circ (S \circ T)$

$dom(W \circ (S \circ T)) = B$ $\quad$ $codom(W \circ (S \circ T)) = B$

$ran(W \circ (S \circ T)) = \{w,x,y,z\} = B$

④ $(T \circ W) \circ S$

$dom((T \circ W) \circ S) = A$ $\quad$ $codom((T \circ W) \circ S) = A$

$ran((T \circ W) \circ S) = \{a,b,c,d\} = A$

⑤ $(V \circ U) \circ R$

$dom((V \circ U) \circ R) = A$ $\quad$ $codom((V \circ U) \circ R) = A$

$ran((V \circ U) \circ R) = \{a,b,c,d\} = A$

⑥ $R \circ (V \circ S)$

$dom(R \circ (V \circ S)) = A \qquad codom(R \circ (V \circ S)) = B$

$ran(R \circ (V \circ S)) = \{w, x, y, z\} = B$

⑦ $(R \circ V) \circ S$

$dom((R \circ V) \circ S) = A \qquad codom((R \circ V) \circ S) = B$

$ran((R \circ V) \circ S) = \{w, x, y, z\} = B$

⑪ $(U \circ R) \circ (T \circ R)$

$dom((U \circ R) \circ (T \circ R)) = A \qquad codom((U \circ R) \circ (T \circ R)) = C$

$ran((U \circ R) \circ (T \circ R)) = \{o, p, q, r\} = C$

10.

(a)

$$M_{R_1} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$R_1^2 = R_1 \circ R_1 = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_1^3 = R_1^2 \circ R_1 = M_{R_1} \odot M_{R_1^2} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1^3 \subseteq R_1$ $\qquad\qquad$ ∴ 관계 R_1은 추이관계이다.

(b)

$$M_{R_2} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$R_1^2 = R_1 \circ R_1 = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^3 = R_1^2 \circ R_1 = M_{R_1} \odot M_{R_1^2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2^3 \not\subseteq R_2$ $\qquad\qquad$ ∴ 관계 R_2은 추이관계가 아니다.

(c)

$$M_{R_3} = \begin{array}{c} \\ a \\ b \\ c \end{array} \begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$R_1^2 = R_1 \circ R_1 = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_1^3 = R_1^2 \circ R_1 = M_{R_1} \odot M_{R_1^2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_3^3 \not\subseteq R_3$ $\qquad\qquad$ ∴ 관계 R_3은 추이관계가 아니다.

(d)

$$M_{R_4} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$R_1^2 = R_1 \circ R_1 = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^3 = R_1^2 \circ R_1 = M_{R_1} \odot M_{R_1^2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_4^3 \subseteq R_4$　　　　　　∴ 관계 R_4은 추이관계이다.

(e)

$$M_{R_4} = \begin{array}{c} \\ a \\ b \\ c \end{array}\!\!\begin{array}{c} \begin{array}{ccc} a & b & c \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \end{array}$$

$$R_1^2 = R_1 \circ R_1 = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$R_1^3 = R_1^2 \circ R_1 = M_{R_1} \odot M_{R_1^2} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$R_5^3 \subseteq R_5$　　　　　　∴ 관계 R_5은 추이관계이다.

11.

(a) 관계 R_1은 추이관계이다.
(b) 관계 R_2는 추이관계이다.
(c) 관계 R_3은 추이관계가 아니다.
(d) 관계 R_4는 추이관계이다.
(e) 관계 R_5는 추이관계가 아니다.

12.

(a)

$S_1 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3), (3,4), (4,2), (4,4)\}$

$S_2 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (4,3), (4,4)\}$

$S_3 = \{(1,1), (1,3), (2,1), (2,2), (3,3), (3,4), (4,2), (4,4)\}$

$S_4 = \{(1,1), (1,3), (2,1), (2,2), (2,4), (3,2), (3,3), (4,1), (4,4)\}$

$S_5 = \{(1,1), (2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4)\}$

(b)

$T_1 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,2), (4,3), (4,4)\}$

$T_2 = \{(1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (3,4), (4,1), (4,2), (4,3)\}$

$T_3 = \{(1,2), (1,3), (2,1), (2,4), (3,1), (3,4), (4,3), (4,2)\}$

$T_4 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (4,4)\}$

$T_5 = \{(1,1), (2,2), (2,3), (3,2), (3,4), (4,3)\}$

(c)

$$M_{R_1}=\begin{array}{c c}
 & \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\
\begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 0\\1 & 0 & 0 & 1\\0 & 1 & 0 & 1\end{bmatrix}
\end{array}$$

$$R_1^2=R_1 \circ R_1=M_{R_1}\odot M_{R_1}=\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 0\\1 & 0 & 0 & 1\\0 & 1 & 0 & 1\end{bmatrix}\odot\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 0\\1 & 0 & 0 & 1\\0 & 1 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 0\\1 & 0 & 0 & 1\\1 & 1 & 0 & 1\\0 & 1 & 1 & 1\end{bmatrix}$$

$$R_1^3=R_1^2 \circ R_1=M_{R_1}\odot M_{R_1^2}=\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 0\\1 & 0 & 0 & 1\\0 & 1 & 0 & 1\end{bmatrix}\odot\begin{bmatrix}1 & 1 & 1 & 0\\1 & 0 & 0 & 1\\1 & 1 & 0 & 1\\0 & 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 0 & 1\\1 & 1 & 1 & 1\\1 & 1 & 1 & 1\end{bmatrix}$$

$$R_1^4=R_1^3 \circ R_1=M_{R_1}\odot M_{R_1^3}=\begin{bmatrix}1 & 1 & 0 & 0\\0 & 0 & 1 & 0\\1 & 0 & 0 & 1\\0 & 1 & 0 & 1\end{bmatrix}\odot\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 0 & 1\\1 & 1 & 1 & 1\\1 & 1 & 1 & 1\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\1 & 1 & 1 & 1\end{bmatrix}$$

$$R_1^*=R_1\cup R_1^2\cup R_1^3\cup R_1^4=M_{R_1}\vee M_{R_1^2}\vee M_{R_1^3}\vee M_{R_1^4}$$

$\therefore\ U_1=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),$
$(4,1),(4,2),(4,3),(4,4)\}$

$$M_{R_2}=\begin{array}{c c}
 & \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\
\begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}0 & 1 & 0 & 1\\1 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}
\end{array}$$

$$R_2^2=R_2 \circ R_2=M_{R_2}\odot M_{R_2}=\begin{bmatrix}0 & 1 & 0 & 1\\1 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}\odot\begin{bmatrix}0 & 1 & 0 & 1\\1 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}$$

$$R_2^3=R_2^2 \circ R_2=M_{R_2}\odot M_{R_2^2}=\begin{bmatrix}0 & 1 & 0 & 1\\1 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}\odot\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}$$

$$R_2^4=R_2^3 \circ R_2=M_{R_2}\odot M_{R_2^3}=\begin{bmatrix}0 & 1 & 0 & 1\\1 & 1 & 0 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}\odot\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\1 & 1 & 1 & 1\\0 & 0 & 1 & 0\\0 & 0 & 1 & 0\end{bmatrix}$$

$$R_2^*=R_2\cup R_2^2\cup R_2^3\cup R_2^4=M_{R_2}\vee M_{R_2^2}\vee M_{R_2^3}\vee M_{R_2^4}$$

$\therefore\ U_2=\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,3),(4,3)\}$

$$M_{R_3}=\begin{array}{c c}
 & \begin{array}{cccc}1 & 2 & 3 & 4\end{array} \\
\begin{array}{c}1\\2\\3\\4\end{array} & \begin{bmatrix}0 & 0 & 1 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 1 & 0 & 0\end{bmatrix}
\end{array}$$

$$R_3^2=R_3 \circ R_3=M_{R_3}\odot M_{R_3}=\begin{bmatrix}0 & 0 & 1 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 1 & 0 & 0\end{bmatrix}\odot\begin{bmatrix}0 & 0 & 1 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 1 & 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0 & 1\\0 & 0 & 1 & 0\\0 & 1 & 0 & 0\\1 & 0 & 0 & 0\end{bmatrix}$$

$$R_3^3 = R_3^2 \circ R_3 = M_{R_3} \odot M_{R_3^2} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$R_3^4 = R_3^3 \circ R_3 = M_{R_3} \odot M_{R_3^3} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \odot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_3^* = R_3 \cup R_3^2 \cup R_3^3 \cup R_3^4 = M_{R_3} \vee M_{R_3^2} \vee M_{R_3^3} \vee M_{R_3^4}$$

$\therefore \; U_3 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

$$M_{R_4} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$R_4^2 = R_4 \circ R_4 = M_{R_4} \odot M_{R_4} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$R_4^3 = R_4^2 \circ R_4 = M_{R_4} \odot M_{R_4^2} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R_4^4 = R_4^3 \circ R_4 = M_{R_4} \odot M_{R_4^3} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$R_4^* = R_4 \cup R_4^2 \cup R_4^3 \cup R_4^4 = M_{R_4} \vee M_{R_4^2} \vee M_{R_4^3} \vee M_{R_4^4}$$

$\therefore \; U_4 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

$$M_{R_5} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$R_5^2 = R_5 \circ R_5 = M_{R_5} \odot M_{R_5} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$R_5^3 = R_5^2 \circ R_5 = M_{R_5} \odot M_{R_5^2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$R_5^4 = R_5^3 \circ R_5 = M_{R_5} \odot M_{R_5^3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$R_5^* = R_5 \cup R_5^2 \cup R_5^3 \cup R_5^4 = M_{R_5} \vee M_{R_5^2} \vee M_{R_5^3} \vee M_{R_5^4}$

$\therefore\ U_5 = \{(1,1), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,2), (4,3), (4,4)\}$

(d)

$S_1 \cup T_1 \cup U_1 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3), (3,4), (4,2), (4,4)\}$
$\cup \{(1,1), (1,2), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,2), (4,3), (4,4)\}\}$
$\cup \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4),$
$(3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$
$= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

관계 $S_1 \cup T_1 \cup U_1$는 반사, 대칭, 추이관계가 성립하므로 동치관계

$\therefore\ O_1 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

$\therefore$ 동치류 $[1] = [2] = [3] = [4] = \{1, 2, 3, 4\}$

$S_2 \cup T_2 \cup U_2 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (4,3), (4,4)\}$
$\cup \{(1,2), (1,4), (2,1), (2,2), (2,4), (3,3), (3,4), (4,1), (4,2), (4,3)\}$
$\cup \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,3), (4,3)\}$
$= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

관계 $S_2 \cup T_2 \cup U_2$는 반사관계만 성립

대칭관계가 성립하도록 하기 위해 $(3,1), (3,2)$ 추가

$\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

그러면 추이관계까지 성립

$\therefore\ O_2 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

$\therefore$ 동치류 $[1] = [2] = [3] = [4] = \{1, 2, 3, 4\}$

$S_3 \cup T_3 \cup U_3 = \{(1,1), (1,3), (2,1), (2,2), (3,3), (3,4), (4,2), (4,4)\}$
$\cup \{(1,2), (1,3), (2,1), (2,4), (3,1), (3,4), (4,3), (4,2)\}$
$\cup \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$
$= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

관계 $S_3 \cup T_3 \cup U_3$는 반사, 대칭, 추이관계가 성립하므로 동치관계

$\therefore\ O_3 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

$\therefore$ 동치류 $[1] = [2] = [3] = [4] = \{1, 2, 3, 4\}$

$S_4 \cup T_4 \cup U_4 = \{(1,1), (1,3), (2,1), (2,2), (2,4), (3,2), (3,3), (4,1), (4,4)\}$
$\cup \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (4,4)\}$
$\cup \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$
$= \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$

관계 $S_4 \cup T_4 \cup U_4$는 반사, 대칭, 추이관계가 성립하므로 동치관계
$\therefore\ O_4 = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4),$
$(4,1), (4,2), (4,3), (4,4)\}$
$\therefore$ 동치류 $[1] = [2] = [3] = [4] = \{1, 2, 3, 4\}$

$S_5 \cup T_5 \cup U_5 = \{(1,1), (2,2), (2,3), (3,2), (3,3), (3,4), (4,3), (4,4)\}$
$\cup \{(1,1), (2,2), (2,3), (3,2), (3,4), (4,3)\}$
$\cup \{(1,1), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,2), (4,3), (4,4)\}$
$= \{(1,1), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,2), (4,3), (4,4)\}$
관계 $S_5 \cup T_5 \cup U_5$는 반사, 대칭, 추이관계가 성립하므로 동치관계
$\therefore\ O_5 = \{(1,1), (2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,2), (4,3), (4,4)\}$
$\therefore$ 동치류 $[1] = \{1\}$, $[2] = [3] = [4] = \{2, 3, 4\}$

13.
(a)
$S_1 = \{(a,a), (a,b), (b,b), (c,c), (c,d), (d,d), (d,e), (e,e)\}$
$S_2 = \{(a,a), (b,b), (b,d), (c,c), (d,d), (d,a), (e,e)\}$
$S_3 = \{(a,a), (b,a), (b,b), (c,b), (c,c), (d,d), (e,e)\}$
$S_4 = \{(a,a), (a,b), (b,b), (b,c), (c,c), (c,d), (d,d), (d,e), (e,a), (e,e)\}$
$S_5 = \{(a,a), (a,b), (b,a), (b,b), (b,c), (c,c), (c,d), (c,e), (d,d), (e,a), (e,b), (e,e)\}$

(b)
$T_1 = \{(a,b), (b,a), (c,c), (c,d), (d,c), (d,e), (e,d)\}$
$T_2 = \{(a,a), (a,d), (b,d), (d,a), (d,b)\}$
$T_3 = \{(a,a), (a,b), (b,a), (b,b), (b,c), (c,b), (c,c), (d,d)\}$
$T_4 = \{(a,b), (a,e), (b,a), (b,c), (c,b), (c,d), (d,c), (d,e), (e,a), (e,d)\}$
$T_5 = \{(a,a), (a,b), (a,e), (b,a), (b,c), (b,e), (c,b), (c,d), (c,e), (d,c), (e,a), (e,b), (e,c), (e,e)\}$

(c)
$U_1 = \{(a,b), (c,c), (c,d), (c,e), (d,e)\}$
$U_2 = \{(a,a), (b,a), (b,d), (d,a)\}$
$U_3 = \{(a,a), (b,a), (b,b), (c,a), (c,b), (c,c), (d,d)\}$
$U_4 = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b), (b,c), (b,d), (b,e), (c,a), (c,b), (c,c),$
$(c,d), (c,e), (d,a), (d,b), (d,c), (d,d), (d,e), (e,a), (e,b), (e,c), (e,d), (e,e)\}$

$U_5 = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b), (b,c), (b,d), (b,e), (c,a), (c,b), (c,c),$
$(c,d), (c,e), (e,a), (e,b), (e,c), (e,d), (e,e)\}$

(d)
$O_1 = \{(a,a), (a,b), (b,a),(b,b), (c,c), (c,d), (c,e), (d,c), (d,d), (d,e), (e,c), (e,d), (e,e)\}$
$\therefore$ 동치류 $[a]=[b]=\{a,b\}$, $[c]=[d]=[e]=\{c,d,e\}$

$O_2 = \{(a,a), (a,b), (a,d), (b,a), (b,b), (b,d), (c,c), (d,a), (d,b), (d,d), (e,e)\}$
$\therefore$ 동치류 $[a]=[b]=[d]=\{a,b,d\}$, $[c]=\{c\}$, $[e]=\{e\}$

$O_3 = \{(a,a), (a,b), (a,c), (b,a), (b,b), (b,c), (c,a), (c,b), (c,c), (d,d), (e,e)\}$
$\therefore$ 동치류 $[a]=[b]=[c]=\{a,b,c\}$, $[d]=\{d\}$, $[e]=\{e\}$

$O_4 = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b), (b,c), (b,d), (b,e), (c,a), (c,b), (c,c),$
$(c,d), (c,e), (d,a), (d,b), (d,c), (d,d), (d,e), (e,a), (e,b), (e,c), (e,d), (e,e)\}$
$\therefore$ 동치류 $[a]=[b]=[c]=[d]=[e]=\{a,b,c,d,e\}$

$O_5 = \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,a), (b,b), (b,c), (b,d), (b,e), (c,a), (c,b), (c,c),$
$(c,d), (c,e), (e,a), (e,b), (e,c), (e,d), (e,e)\}$
$\therefore$ 동치류 $[a]=[b]=[c]=[e]=\{a,b,c,d,e\}$, $[d]=\{\ \}$

14.

관계 R_1에 대해,

$$R_1 = M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

주대각 원소가 모두 1이다.
$\therefore$ 관계 R_1은 반사관계이다.

주대각 원소를 기준으로 마주보는 원소들의 값이 서로 같다.
$\therefore$ 관계 R_1은 대칭관계이다.

$$R_1^2 = R_1 \circ R_1 = M_{R_1} \odot M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_1^3 = R_1^2 \circ R_1 = M_{R_1} \odot M_{R_1^2} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$R_1^4 = R_1^3 \circ R_1 = M_{R_1} \odot M_{R_1^3} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_1^4 \subseteq R_1$이다.

$\therefore$ 관계 R_1은 추이관계이다.

$\therefore$ 관계 R_1은 반사, 대칭, 추이관계이다.

$\therefore$ 관계 R_1은 동치관계이다.

관계 R_2에 대해,

$$R_2 = M_{R_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

주대각 원소가 모두 1이다.

$\therefore$ 관계 R_2는 반사관계이다.

주대각 원소를 기준으로 마주보는 원소들의 값이 서로 다르다.

$\therefore$ 관계 R_2는 반대칭관계이다.

$$R_2^2 = R_2 \circ R_2 = M_{R_2} \odot M_{R_2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = R_2^2 \circ R_2 = M_{R_2} \odot M_{R_2^2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^4 = R_2^3 \circ R_2 = M_{R_2} \odot M_{R_2^3} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^5 = R_2^4 \circ R_2 = M_{R_2} \odot M_{R_2^4} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2^5 \subseteq R_2$이다.

$\therefore$ 관계 R_2는 추이관계이다.

$\therefore$ 관계 R_2는 반사, 반대칭, 추이관계이다.

$\therefore$ 관계 R_2는 부분순서관계이다.

관계 R_3에 대해,

$$R_3 = M_{R_3} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

주대각 원소가 모두 1이다.
$\therefore$ 관계 R_3는 반사관계이다.

주대각 원소를 기준으로 마주보는 원소들의 값이 서로 같다.
$\therefore$ 관계 R_3는 대칭관계이다.

$$R_3^2 = R_3 \circ R_3 = M_{R_3} \odot M_{R_3} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$R_3^3 = R_3^2 \circ R_3 = M_{R_3} \odot M_{R_3^2} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$R_3^4 = R_3^3 \circ R_3 = M_{R_3} \odot M_{R_3^3} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$R_3^5 = R_3^4 \circ R_3 = M_{R_3} \odot M_{R_3^4} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \odot \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$R_3^5 \subseteq R_3$이다.
$\therefore$ 관계 R_3는 추이관계이다.

$\therefore$ 관계 R_3는 반사, 대칭, 추이관계이다.
$\therefore$ 관계 R_3는 동치관계이다.

관계 R_4에 대해,

모든 원소 $a \in D$에 대해 $a|a$는 항상 성립하므로 $(a, a) \in R_4$
$\therefore$ 관계 R_4는 반사관계이다.

어떤 원소 $a, b \in D$에 대해 $a \neq b$일 때, $b|a$이면 $a = kb(k \in N)$이고 $b = \frac{a}{k}$이다. 따라서 $b < a$이므로 $a \nmid b$이다. 그러므로 $(a, b) \in R_4$이면 $(b, a) \not\in R_4$이다.
$\therefore$ 관계 R_4는 반대칭관계이다.

어떤 원소 $a, b, c \in D$에 대해 $b|a$이고 $c|b$이면 $a = kb(k \in N)$, $b = lc(l \in N)$이다. 그러면 $b = lc$를 $a = kb$에 대입하면 $a = klc$이므로 $c|a$가 성립한다.

그러므로 $(a, b) \in R_4$이고 $(b, c) \in R_4$이면 $(a, c) \in R_4$이다.

$\therefore$ 관계 R_4는 추이관계이다.

$\therefore$ 관계 R_4는 반사, 반대칭, 추이관계이다.

$\therefore$ 관계 R_4는 부분순서관계이다.

관계 R_5에 대해,

모든 원소 $a \in Z$에 대해, $a = a$는 항상 성립하므로 $(a, a) \in R_5$

$\therefore$ 관계 R_5는 반사관계이다.

어떤 원소 $a, b \in Z$에 대해 $a = b$이면 $b = a$이므로 $(a, b) \in R_5$이면 $(b, a) \in R_5$

단, 관계 R_5의 순서쌍을 구성하는 원소는 $a = b$

$\therefore$ 관계 R_5는 대칭관계이면서 반대칭관계이다.

어떤 원소 $a, b, c \in Z$에 대해 $a = b$이고 $b = c$이면 $a = c$이므로 $(a, b) \in R_5$이고 $(b, c) \in R_5$이면 $(a, c) \in R_5$

$\therefore$ 관계 R_5는 추이관계이다.

$\therefore$ 관계 R_5는 반사, 대칭, 반대칭 추이관계이다.

$\therefore$ 관계 R_5는 동치관계이면서 부분순서관계이다.

관계 R_6에 대해,

모든 원소 $a \in N$에 대해 $a \leq a$는 항상 성립하므로 $(a, a) \in R_6$

$\therefore$ 관계 R_6는 반사관계이다.

어떤 원소 $a, b \in N$에 대해 $a \leq b$이면 $b \leq a$는 성립하지 않는다. 그러므로 $(a, b) \in R_6$이면 $(b, a) \not\in R_6$

$\therefore$ 관계 R_6는 반대칭관계이다.

어떤 원소 $a, b, c \in N$에 대해 $a \leq b$이고 $b \leq c$이면 $a \leq c$

$\therefore$ 관계 R_6는 추이관계이다.

$\therefore$ 관계 R_6는 반사, 반대칭, 추이관계이다.

$\therefore$ 관계 R_6는 부분순서관계이다.

15.

동치관계 R_1의 동치류 : $[1] = [2] = \{1, 2\}$ $[3] = [4] = \{3, 4\}$ $S_1 = \{\{1, 2,\}, \{3, 4\}\}$

동치관계 R_3의 동치류 : $[1] = [3] = [5] = \{1, 3, 5\}$ $[2] = [4] = \{2, 4\}$ $S_3 = \{\{1, 3, 5\}, \{2, 4\}\}$

동치관계 R_5의 동치류 : 모든 원소 $a \in Z$에 대해 $[a] = \{a\}$

16.

(a)

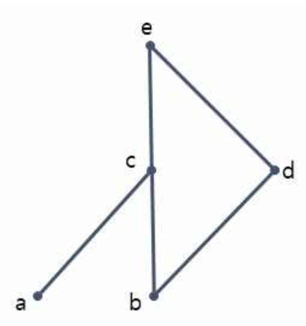
부분순서관계 R_2	
비교가능	비교불가능
a와 a, c, e	a와 b, d
b와 d, c, d, e	b와 a
c와 c, e	c와 a, b, d
d와 d, e	d와 a, b, c
e와 e	e와 a, b, c, d

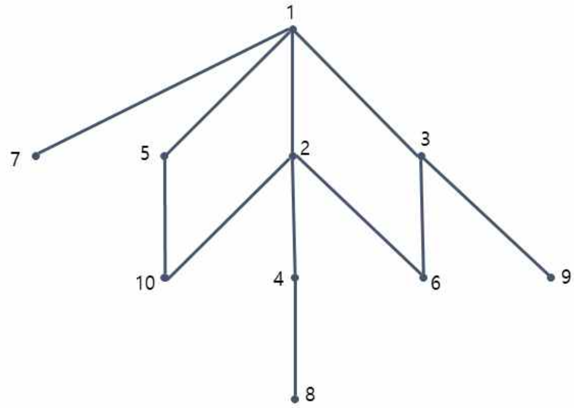
부분순서관계 R_4	
비교가능	
1과 1, 2, 3, 4, 5, 6, 7, 8, 9, 10	6과 1, 2, 3, 6
2와 1, 2, 4, 6, 8, 10	7과 1, 7
3과 1, 3, 6, 9	8과 1, 2, 4, 8
4와 1, 2, 4, 8	9과 1, 3, 9
5와 1, 5, 10	10과 1, 2, 5, 10
비교불가능	
1은 없음	6과 4, 5, 7, 8, 9, 10
2와 3, 5, 7, 9	7과 2, 3, 4, 5, 6, 8, 9, 10
3과 2, 4, 5, 8, 10	8과 3, 5, 6, 7, 9, 10
4와 3, 5, 6, 7, 9, 10	9와 2, 4, 5, 6, 7, 8, 10
5와 2, 3, 4, 6, 7, 8, 9	10과 3, 4, 6, 7, 8, 9

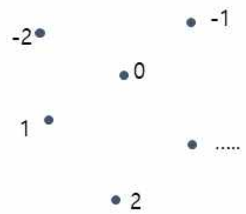
부분순서관계 R_5의 모든 원소 $a \in Z$는 자기자신 a와 비교 가능, 다른 원소와는 비교 불가능

부분순서관계 R_6의 모든 원소는 서로 비교 가능 ∴ 완전순서관계

(b)

<table>
<tr><td>관계 R_2에 대한 하세도표</td><td>관계 R_4에 대한 하세도표</td></tr>
<tr><td>


</td><td>


</td></tr>
<tr><td>관계 R_5에 대한 하세도표</td><td>관계 R_6에 대한 하세도표</td></tr>
<tr><td>


</td><td>


</td></tr>
</table>

(c)

관계 R_2의 극대원소 : e / 극소원소 : a, b / 최대원소 : e / 최소원소 : 없음

관계 R_4의 극대원소 : 1 / 극소원소 : 6, 7, 8, 9, 10 / 최대원소 : 1 / 최소원소 : 없음

관계 R_5의 극대원소 : 없음 / 극소원소 : 없음 / 최대원소 : 없음 / 최소원소 : 없음

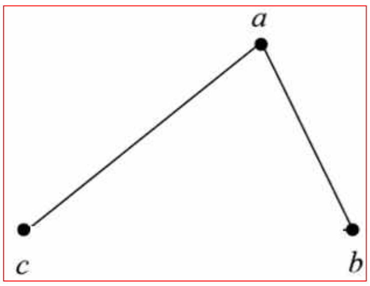
관계 R_6의 극대원소 : 1 / 극소원소 : 자연수 범위의 최댓값

최대원소 : 1 / 최소원소 : 자연수 범위의 최댓값

17.

<<문제 수정>> ②번 그림을 아래 그림으로 수정합니다.

②



(a)
① 극대원소 : a / 극소원소 : f, h, i / 최대원소 : a / 최소원소 : 없음
② 극대원소 : a / 극소원소 : c, d / 최대원소 : a / 최소원소 : 없음
③ 극대원소 : a, b, c / 극소원소 : g, h / 최대원소 : 없음 / 최소원소 : 없음
④ 극대원소 : a, d / 극소원소 : e / 최대원소 : 없음 / 최소원소 : e

(b)
① 집합 $A=\{a,b,c,d,e,f,g,h,i\}$에 대한 관계
$\{(a,a),(b,a),(b,b),(c,c),(c,a),(d,a),(d,b),(d,d),(e,a),(e,b),(e,e),(f,a),(f,c),(f,f),$
$(g,a),(g,a),(g,b),(g,d),(g,g),(h,a),(h,b),(h,e),(h,h),(i,a),(i,b),(i,d),(i,g),(i,i)\}$

② 집합 $A=\{a,b,c\}$에 대한 관계 $\{(a,a),(b,a),(b,b),(c,a),(c,c)\}$

③ 집합 $A=\{a,b,c,d,e,f,g,h,i\}$에 대한 관계
$\{(a,a),(b,b),(c,c),(d,b),(d,d),(e,c),(e,e),(f,b),(f,c),(f,d),(f,f),(g,a),(g,c),(g,e),(g,g),$
$(h,b),(h,c),(h,d),(h,f),(h,h),(h,i),(i,b),(i,i)\}$

④ 집합 $A=\{a,b,c,d,e\}$에 대한 관계
$\{(a,a),(b,a),(b,b),(b,d),(c,a),(c,c),(c,d),(d,d),(e,a),(e,b),(e,c),(e,d),(e,e)\}$

18.
(a)
① 극대원소 : 자연수 집합 $\mathbb{N}$에서의 최댓값 극소원소 : 1
최대원소 : 자연수 집합 $\mathbb{N}$에서의 최댓값 최소원소 : 1
② 극대원소 : i / 극소원소 : h / 최대원소 : i / 최소원소 : h
③ 극대원소 : g, h, i / 극소원소 : a, b, c / 최대원소 : 없음 / 최소원소 : 없음
④ 극대원소 : 8, 9, 10, 11, 12, 13, 14, 15 / 극소원소 : 1 / 최대원소 : 없음 / 최소원소 : 1

(b)
① 자연수 집합 $\mathbb{N}$에 대한 관계 $\{(a,b)\in\mathbb{N}\times\mathbb{N} \mid a \le b\}$
② 집합 $A=\{a,b,c,d,e,f,g,h,i\}$에 대한 관계
$\{(a,a),(a,d),(a,f),(a,i),(b,b),(b,d),(b,i),(c,c),(c,f),(c,i),(d,d),(d,i),(e,a),(e,c),(e,d),$
$(e,e),(e,f),(e,i),(f,f),(f,i),(g,a),(g,b),(g,d),(g,f),(g,g),(g,i),(h,a),(h,b),(h,c),(h,d),$
$(h,e),(h,f),(h,g),(h,h),(h,i),(i,i)\}$

③ 집합 $A=\{a,b,c,d,e,f,g,h,i\}$에 대한 관계
$\{(a,a),(a,d),(a,g),(b,b),(b,e),(b,h),(c,c),(c,f),(c,i),(d,d),(d,g),(e,e),(e,h),$
$(f,f),(f,i),(g,g),(h,h),(i,i)\}$

④ 집합 $A=\{a \mid a \le 15, a\in N\}$에 대한 관계 $\{(a,b)\in A\times A \mid a \mid b\}$